Financial Opacity and Bank’s Asset Portfolio Strategy

JOB MARKET PAPER

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Abstract

This paper studies the optimal contract and portfolio decisions of banks in an opaque banking system where consumers have limited information about each bank’s asset portfolio and its ability to repay debt. This paper is able to explain the bank asset portfolio heterogeneity observed in the 2007-2009 financial crisis, in which some banks incurred large losses from risky assets while other banks had abundant liquidity and made profitable purchases and acquisitions. In this banking system, inefficient risk-taking is undetectable by consumers until a bank incurs losses. Upon learning about such losses, consumers make a run on the affected bank, which becomes bankrupt and begins to liquidate assets. In equilibrium, as banks are leveraged, risk-taking and potential bankruptcies of a small number of banks are generally unavoidable. However, these bankruptcies indirectly prevent more banks from engaging in risk-taking. Because healthy banks can profit from the asset sales and liquidations of risk-taking banks, an increasing number of bankruptcies will allow healthy banks to offer more favorable contracts to consumers, making further risk-taking less attractive. Therefore the banking sector is endogenously divided into risk-taking and healthy banks. The banking model I develop has an optimal level of capital requirement. A capital requirement that is too high or too low is not optimal for consumer welfare.

Keywords: Banking, Financial Markets, Liquidity, Government Policy
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1 Introduction

This paper is motivated by two salient features observed in the 2007-2009 financial crisis. First, financial institutions displayed a large degree of heterogeneity in their asset portfolios and performance. I divide financial institutions into two groups based on their pre-crisis asset portfolios. Some financial institutions invested heavily in risky assets prior to the crisis. When these assets incurred losses, the investing institutions became distressed or bankrupt. Meanwhile, other financial institutions did not invest much in risky assets and remained relatively healthy during the crisis.

The second feature was the transfer of illiquid assets from distressed banks to healthy banks at record low prices. Upon learning about the losses, consumers started withdrawing money from distressed banks, which had to sell large amounts of illiquid assets to obtain liquidity. Massive simultaneous asset sales by multiple banks and limited market liquidity led to severe mispricing of illiquid assets. The deep discounts on the prices of illiquid assets provided opportunities for healthy banks with abundant liquidity to make profitable purchases and acquisitions.

After the crisis, the Financial Crisis Inquiry Report (FCIC) concluded that failures of risk management were a key cause of the crisis. High leverage, risky investments, and lack of transparency were significant factors that contributed to the crisis. As stated in the FCIC (2011), “Leverage ratios were as high as 40 to 1 and the leverage was often hidden..... The heavy debt taken on by some financial institutions was exacerbated by the risky assets they were acquiring with that debt..... The dangers of this debt were magnified because transparency was not required or desired.” In other words, some financial institutions operated with high leverage and loaded up with risky securities from the mortgage market, but neither creditors nor authorities knew the precise debt obligations and asset positions of these institutions.

According to the corporate credit default swap (CDS) data, most major financial institutions had similar levels of perceived risk prior to the crisis. There was no indication that some institutions were significantly riskier than others. But after information about losses was revealed, market participants stopped providing short-term financing and depositors and clients started withdrawing. The CDS of some institutions then spiked. From the time asset positions were taken until the losses were reported, the asset portfolios were relatively non-transparent.

This paper studies the optimal contract and portfolio decisions of banks in an opaque banking system where consumers have limited information about each bank’s asset portfolio and its ability to repay debt. I develop a realistic banking model that is able to account for the asset portfolio het-

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1As documented in FCIC (2011), by June 2007, AIG had $79 billion derivatives exposure to mortgage-related securities, the majority of which were Alt-A and subprime. AIG hedged no more than $150 million of its subprime exposure, while Goldman Sachs hedged aggressively by buying CDS protection on AIG to counterbalance the risk of potential AIG failure or a collapsing subprime market. By April 2007, Bear Stearns’s hedge funds had about 60% collaterals in subprime mortgage-backed CDO’s. Lehman Brothers had $111 billion in real estate holdings and WaMu was a leader in subprime lending with 56% of loan portfolio in subprime loans, option-ARM, and home equity loans.

2JP Morgan and Bank of America were considerably healthier with much smaller subprime exposure. Warren Buffet’s Berkshire Hathaway was a major liquidity provider in the insurance sector during the crisis.

3For a review of literature documenting the fire sales of 2007-2009 financial crisis, see Shleifer and Vishny (2011).

4As Warren Buffet wrote in 2001, “After all, you only find out who is swimming naked when the tide goes out.”

5See appendix for the graph of 5-year corporate CDS for major financial institutions.
erogeneity and the acquisition dynamics observed during the 2007-2009 financial crisis. I also study
the policy implications of capital requirements on risk-taking and welfare. The model generates
positive descriptions of the real world and provides interesting and important predictions.

The model, which is based on Allen and Gale (2004a), has three periods 0, 1, and 2. Consumers
receive endowment only from period 0 and face preference shocks for consumption in the next
two periods. A consumer can deposit his endowment into a bank in exchange for a consumption
contract that provides a consumption payment in period 1 or period 2. Banks compete by offering
the most attractive consumption contracts to consumers. There are three types of assets: short,
long, and risky. The short asset is a one-period liquid investment that can be invested in periods 1
and 2. The long asset is an illiquid investment that takes two periods to mature. It can be invested
in period 0 and it generates a higher return than the short asset. The risky asset is also an illiquid
investment. It generates a more attractive return than the long asset in good economic times and
it loses principal in bad times. The risky asset is considered inefficient because its expected return
is smaller than that of the other two assets.

The banks are operated by risk-neutral bankers who receive endowment at period 0 and consume
at period 2. A banker allocates part of his endowment as equity capital into a bank and he is
protected by limited liability. The remaining part of his endowment is invested in an investment
opportunity outside of the banking sector. After attracting consumers, the bankers invest in a
portfolio of liquid and illiquid assets using consumer deposits and equity capital, and they use the
investment returns to pay consumer contracts. The remaining resources are paid to the bankers
at the end of period 2. Banks face idiosyncratic liquidity shocks as there is uncertainty about the
fraction of consumers who will withdraw early at each bank. Bankers make contract and portfolio
decisions designed to maximize their total expected payoff. Financial opacity exists as consumers
have limited information about each bank’s asset portfolio and its ability to repay debt. Investment
in the inefficient risky asset is undetectable by consumers at period 0 until it causes the banks to
incur losses, which will be revealed at period 1. Upon learning about such losses, consumers make
a run on the affected banks, which become bankrupt and begin to liquidate assets.

I begin with the analysis of the optimal asset portfolio and consumption plan that a social
planner would implement. I use the constrained efficiency concept by only allowing the planner
to offer the same type of contract as the banks. The planner’s allocation involves no investment
in the risky asset, an optimal combination of short and long asset investments, and a smoothed
consumption plan across two periods for consumers. In addition, the planner uses an optimal level
of capital to generate more resources for consumers.

I then analyze the banking equilibrium in which banks offer consumption contracts to consumers.
When financial opacity does not exist and consumers have perfect information about a bank’s asset
portfolio, there cannot be investment in the inefficient risky asset. Contracts are fully deliverable
at all times and consumers will choose the contract that provides the highest expected utility.
In this non-opaque case, the banking equilibrium is a risk-free equilibrium and its allocation is
constrained efficient. Specifically, competition among banks forces them to choose the optimal
asset portfolio and offer contracts that maximize consumer utility. In other words, bankers choose the same socially optimal levels of capital, short asset, and long asset that a social planner would choose. The optimal contract provides as much returns to consumers as possible, giving bankers the same payoff they would obtain by investing the equity capital elsewhere.

When financial opacity exists and consumers have limited information about each bank’s asset portfolio, it is generally inevitable that a small number of banks will engage in inefficient risk-taking and face potential bankruptcy. The risk-free equilibrium no longer exists and the constrained efficient allocation cannot be supported. Intuitively, as banks are leveraged, some banks can profit from risky investments by offering consumers the same contract as non-risk-taking banks. Such undetectable risky investments allow the banks to gain additional profits in good economic times. When risky investments incur losses in bad times, the risk-taking banks will not have sufficient resources to pay all consumer contracts and all consumers will withdraw early. The risk-taking banks become bankrupt and start selling illiquid assets. The liquidation proceeds are paid evenly to all consumers and the bankers’ losses are bounded above by their initial capital contributions. As long as the risky asset generates a sufficiently attractive return in good times, profitable risk-taking by a small number of banks can always exist in the banking system.

The main result of this paper is that the model is able to generate bank asset portfolio heterogeneity. I show that under financial opacity, in equilibrium there exists a banking system with two types of banks: safe banks and risky banks. Safe banks do not invest in the risky asset and they offer a contract that maximizes consumer utility. Risky banks invest in the inefficient risky asset but they offer the same contract as safe banks. There is no ex-ante heterogeneity among banks, but the banking sector is endogenously divided into risky and safe banks. Consumers cannot detect inefficient risk-taking when selecting the contract, and therefore may suffer from a low consumption payoff in bad times if they are with a risky bank. Such portfolio and performance heterogeneity was observed in the 2007-2009 financial crisis. Some banks incurred large losses from risky assets while others had abundant liquidity and made profitable purchases and acquisitions.

The novel result on the endogenous division of the banking system is contributed by the general equilibrium perspective of the model. When financial opacity exists, it is generally inevitable that a small number of banks will make inefficient risky investments and face potential bankruptcy. However, the bankruptcies of these risk-taking banks serve indirectly to prevent more banks from engaging in risk-taking.

Intuitively, as more banks engage in risk-taking, the size of potential bankruptcies and liquidations increases. In bad economic times, bank runs force these risk-taking banks to sell large amounts of illiquid assets at the same time. Massive asset sales and limited market liquidity lead to low prices for illiquid assets. These deeply discounted asset prices allow safe, healthy banks to make profitable acquisitions using available liquidity. Driven by competition and consumption smoothing motive, safe banks can then offer contracts with higher consumption payments in all economic states. This increase in overall contract payments reduces the additional profits that a risk-taking bank could obtain in good times (the only times that risk-taking bankers can receive
payoffs). As a result, risk-taking becomes less profitable and eventually undesirable as the number of risky banks increases.

In short, an undetectable risk-taking strategy can be profitable for a small number of risky banks if they mimic the contract offered by safe banks. Since safe banks can profit from the asset sales and liquidations of bankrupt institutions, an increasing number of bankruptcies will allow safe banks to offer more attractive contracts to consumers. Mimicking an increasingly expensive contract makes further risk-taking less favorable.

The existence of undetected inefficient risk-taking generates a market failure and provides a justification for government intervention. In the banking model described herein, there exists an optimal level of capital requirement. A capital requirement that is too high or too low is not optimal for consumer welfare. Increasing the capital requirement has two impacts on welfare. On one hand, a higher capital requirement improves welfare by reducing the number of banks that engage in inefficient risk-taking. On the other hand, due to the increasing opportunity cost of using capital, a higher capital requirement decreases welfare by causing banks to expend more resources for using capital and lower their consumption payments to consumers. Specifically, non-risk-taking banks are forced to use additional capital at an expensive cost that cannot be fully financed within the banking sector. In order for banks to break even, the consumption contracts offered to consumers have to be reduced.

The overall impact on welfare of increasing the capital requirement depends on the relative magnitude of each impact. Generally, increasing the capital requirement at low levels can potentially improve consumer welfare but capital requirement that is too high can reduce it. It may not be optimal to completely eliminate risk-taking by imposing an excessively high capital requirement ratio. Although this would eliminate inefficient risk-taking, it would force safe banks to operate with highly costly capital. Safe banks would not be able to finance the usage cost of this capital within the banking sector and additional resources would have to be taken from consumers to pay for the usage of capital. As a result, consumer contracts would have to be cut down substantially for banks to break even and aggregate welfare would be reduced.

1.1 Related Literature

This paper is closely related to the analysis of financial intermediaries by Allen and Gale (2004a, 2004b). In those papers, authors develop a general equilibrium banking model and analyze the efficiency of banking equilibriums. In those models, risky assets are assumed to be socially efficient and beneficial for consumer welfare. Because of competition, the benefits of risk-taking are passed onto the consumers through financial intermediaries. The banking equilibrium allocation is generally efficient or constrained efficient with complete markets, even in the event of default. There is no market failure and all consumers receive the same level of utility.6

The novelty of this paper is that I explore the impact of inefficient risk-taking. There is a

6In Allen, Carletti, and Gale (2009), the banking equilibrium allocation is not constrained efficient because of uninsurable aggregate liquidity shocks. Yet consumers of all banks receive the same level of utility.
risky asset that is assumed to be socially inefficient and dominated by other assets in the economy. Because of leverage and limited liability, some banks have incentives to invest in the risky asset, which could trigger runs and defaults in bad economic times. Consumers with risk-taking institutions receive strictly lower level of utility than consumers with non-risk-taking institutions. There exists a market failure and provides a justification for government intervention.

The overall majority of existing banking literature has focused on banking models with a partial equilibrium perspective or without interactions between banks. Different notions of moral hazard and risk-taking have been adopted into various setups and have led to inefficiencies of the banking system. Hellmann, Murdock, and Stiglitz (2000) show that a higher capital requirement can reduce the franchise values of banks and the incentive of prudent behavior. Adrian and Shin (2011) use a contracting model with moral hazard to argue that monetary loosening can increase risk-taking. Farhi and Tirole (2012) show that excessive maturity mismatch can emerge with the anticipation of ex-post bailout policy. Acharya and Viswanathan (2011) use a moral hazard setup to show the existence of excessive risk-taking and cash-in-the-market pricing. Dell’Ariccia, Laeven, and Marquez (2010) develop a theoretical framework to study the impact of monetary policy on risk-taking through various channels. Brusco and Castiglionesi (2007) show that there is trade-off between liquidity coinsurance and excessive risk-taking in a model with moral hazard and contagion. Inefficiency can also be caused by noisy signals. Goldstein and Pauzner (2005) build on Diamond and Dybvig (1983) and derive a unique bank-run equilibrium using the global games framework. Panic-based bank runs are initiated by adverse news and are results of coordination failures. In addition, there is a line of literature that analyzes on the role of liquidity in determining asset prices based on general equilibrium frameworks.7

These models have important and interesting implications either on the behavior of a single representative bank or on the outcome of banking system. However, most of them cannot account for the heterogeneity in bank asset portfolio and performance without assuming ex-ante heterogeneity. In particular, it is not possible to generate the situation where banks choose different asset holdings but have the same level of perceived risk.

A large literature on bank risk-taking has focused on the role of deposit insurance8 and other mechanisms that distort banks from choosing the socially optimal decisions. Some earlier literature such as Keeley and Furlong (1990) build on the existence of deposit insurance to study the impact of capital requirements on bank risk-taking. Other works such as Kim and Santomero (1988) and Blum (1999) show that a higher capital requirement can increase bank risk-taking. Repullo (2004) shows that capital requirements can be effective in ensuring a prudent equilibrium but may be costly to the banks. Boyd and De Nicolo (2005) show that as competition declines, bankruptcy risk can be higher as banks charge higher loan rates. Empirically, Laeven and Levine (2009) find supports for the arguments that banks with dominating equity holders have strong incentives to induce bank’s managers to increase risk taking. Comparing to those papers, this paper does not assume the

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7For examples, see Gorton and Huang (2004), Bolton, Santos, and Scheinkman (2011).
8For theoretical and empirical examples, see Keeley (1990) and Demirgüç-Kunt and Detragiache (2002).
existence of deposit insurance or other exogenous factors that affect incentives. Risk-taking takes
the form of asset substitution and the risks are shifted to the consumers.

So far no banking literature has studied the impact of inefficient risk-taking in a general equilib-
rium banking framework. By incorporating an inefficient risky asset into the banking system, this
paper is able to explain the bank asset portfolio heterogeneity as observed during the 2007-2009
financial crisis. Inefficient risk-taking can lead to potential bankruptcies of a subset of the financial
institutions, severe mispricing of illiquid assets in bad economic times, and welfare losses to con-
sumers. In this sense, this paper also contributes to standard macroeconomic general equilibrium
models in which financial intermediaries generally play neutral roles and cannot cause undesirable
outcomes. As observed in the real world, the behaviors of financial institutions can have large
impacts on consumer welfare.

The rest of the paper is organized as follows. Section 2 describes the model environment and
the constrained efficient allocation. Section 3 analyzes the banking equilibrium without and with
financial opacity. It is shown that there exists a banking equilibrium with two types of banks
when financial opacity exists. Section 4 analyzes the impact of capital requirements on the banking
equilibrium and welfare. Section 5 concludes the paper.

2 The Model

The model builds on the banking models developed in Allen and Gale (2004a, 2004b) and Allen,
Carletti, and Gale (2009). There are three dates \( t = 0, 1, 2 \) and a single good at each date. The
good is used for consumption and investment at each date.

At \( t = 0 \) there is a continuum of ex-ante identical depositors (consumers) in \([0, 1]\), where each
point in the interval is a different agent. The measure of entire set of consumers is normalized to 1,
and the measure of the fraction of consumers in any subset is its Lebesgue measure. Each consumer
has an endowment of 1 unit of good at \( t = 0 \) and no endowment at \( t = 1 \) and \( t = 2 \). Consumers
are subject to idiosyncratic preference shocks. A consumer is either an early consumer, who only
values consumption at \( t = 1 \). Or a consumer can be a late consumer, who only values consumption
at \( t = 2 \). The details about the uncertainty in consumer preferences will be described later. The
utility of consumption is represented by a utility function \( u(c) \) with \( u'(c) > 0 \) and \( u''(c) < 0 \).

There is a large number of banks in the economy represented by the interval \([0, 1]\). At \( t = 0 \),
banks offer consumption contracts and collect deposits from consumers. Each bank has a large
number of consumers and the measure of each bank’s consumers is zero. Banks face capital re-
quirement ratio \( \bar{e} > 0 \), which is set exogenously by the banking regulatory authority. For each unit
of deposit collected from consumers, a bank must provide at least \( \bar{e} \) units of equity capital.

Banks raise equity capital from bankers (outside capitalists). There is a continuum of bankers
in \([0, N]\), where each point in the interval is a different banker. The measure of entire set of bankers
is normalized to \( N > 1 \). At \( t = 0 \) each banker has \( w > 0 \) units of good as endowment. Bankers are
risk neutral and they value consumption at \( t = 2 \). Given any capital requirement \( \bar{e} > 0 \), if bankers
want to participate in the banking sector, they allocate \( e \in [\bar{e}, w] \) units of good as equity capital into the banks. The remaining resources \((w - e)\) are invested in an investment opportunity outside of banking sector.

The outside investment opportunity is represented by a production function \( F_O(\hat{w}) \), \( \hat{w} \in [0, w] \) through which \( \hat{w} \) units of good invested at \( t = 0 \) can generate \( F_O(\hat{w}) \) units of good at \( t = 2 \). The function \( F_O(\hat{w}) \) satisfies \( F_O'(\hat{w}) > 1 \) and \( F_O''(\hat{w}) < 0 \), \( \forall \hat{w} > 0 \). For a banker providing a capital level of \( e \), denote \( K(e) \) as the opportunity cost of using capital

\[
K(e) = F_O(w) - F_O(w - e)
\]

In order to be willing to participate, the banker must receive at least \( K(e) \) units of good at \( t = 2 \) from the bank to cover the cost of capital. Using capital is increasingly costly as \( K'(e) > 1, K''(e) > 0 \).

### 2.0.1 Assets

There are three assets: short-term safe asset, long-term safe asset, and long-term risky asset. The short-term safe asset (the short asset) is a risk-free storage technology: one unit of good invested at date \( t \) produces a return of one unit of good at date \( t + 1 \) for \( t = 0,1 \). The long-term safe asset (the long asset) is a safe investment technology: \( z \) units invested at \( t = 0 \) can generate \( F(z) \) units of good at \( t = 2 \). The function \( F(z) \) satisfies

\[
F'(z) > 1 \quad \text{and} \quad F''(z) < 0 \quad \forall z > 0
\]

Economically speaking, long asset has diminishing marginal returns, but it is always more productive than short asset. As a result, average return of investment in long asset \( r_F(z) = \frac{F(z)}{z} > 1, \forall z > 0 \).

The long-term risky asset (the risky asset) takes one unit of investment at \( t = 0 \) and it generates a random return of \( R \) units of good at \( t = 2 \), where \( R \) is a non-negative random variable whose value depends on aggregate economic state \( i \in \{G, B\} \) (Good) or \( i = B \) (Bad)

\[
R = \begin{cases} 
R_G > F'(z) & \forall z \quad \text{with probability } \pi, \\
R_B < 1 & \text{with probability } 1 - \pi.
\end{cases}
\]

Assume \( E[R] = \pi R_G + (1 - \pi) R_B < 1 \). Investment in the risky asset is relatively inefficient than investments in short asset and long asset. The risky asset generates an attractive return \( R_G \) in the \( G \) state but loses principal in the \( B \) state. An example of risky asset is the subprime mortgage security. The aggregate state \( i \in \{G, B\} \) is realized at \( t = 1 \) but the risky asset remains illiquid until \( t = 2 \).

Banks have access to all three assets while consumers do not have access to any asset. Consumers can choose to store their endowments privately. Private storage by consumers generates the same

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9. \( K'(e) = F'_O(w - e) > 1 \) and \( K''(e) = -F''_O(w - e) > 0 \) as \( F'_O(\hat{w}) < 0 \).
10. This setup is similar to many banking literatures that assume a leading economic indicator is first known but the asset remains illiquid. For example, see Allen and Gale (2007).
return as the short asset. One unit stored at date $t$ produces one unit at date $t+1$ for $t = 0, 1$. Consumers cannot gather together to trade goods. A consumer either deals with a bank or consumes his endowment via private storage.

### 2.0.2 Liquidity and Preference Shocks

At $t = 1$ each bank faces a bank-specific idiosyncratic liquidity shock $\lambda$, which represents the fraction of early consumers at the bank. The random variable $\lambda$ has a two-point support and can be expressed as

$$\lambda = \begin{cases} 
\lambda_H & \text{with probability } \frac{1}{2}, \\
\lambda_L & \text{with probability } \frac{1}{2},
\end{cases}$$

where $0 < \lambda_L < \lambda_H < 1$.

At the banks with liquidity shock $j \in \{H, L\}$, $\lambda_j$ fraction of consumers are early consumers who will consume at $t = 1$. The other $(1 - \lambda_j)$ fraction of consumers are late consumer who will consume at $t = 2$. According to the law of large numbers, since there is a large number of banks, at $t = 1$ half of the banks will face high liquidity shock $\lambda_H$ and half of the banks will face low liquidity shock $\lambda_L$. There is no uncertainty about the proportion of each type of consumers in the economy. The fraction of early consumers in the economy is fixed and equal to $(\frac{1}{2}\lambda_H + \frac{1}{2}\lambda_L)$.

A consumer’s type depends on which bank he is with. In other words, a consumer’s preference shock depends on the bank-specific liquidity shock at his own bank. If a consumer is with a $\lambda_j$ shock bank, then he is an early consumer with probability $\lambda_j$. With probability $(1 - \lambda_j)$, he is a late consumer.\(^\text{11}\) The graph below describes the preference shock from a consumer’s perspective.

![Preference Shock Diagram](chart.png)

Since a consumer does not know his preference regarding the time of consumption and does not have access to asset market, the uncertainty and lack of market access generate a role for the banks to solve the mismatch between asset maturity and time preferences. In this model, bankers maximize their payoffs by making decisions in two steps. First, bankers decide what consumption contract to offer to attract consumers. Second, if the bank has successfully attracted consumers, the bankers decide what capital level and asset portfolio to choose.

\(^\text{11}\)Since the law of large numbers applies within each bank, this setup ensures consistency and distributes early consumers into banks.
2.0.3 Consumption Contract

The consumption contract can be made contingent on time period and liquidity shock \( j \in \{H, L\} \), which will be realized at \( t = 1 \). Since aggregate state \( i \in \{G, B\} \) is not observable, contract cannot be contingent on \( i \). All contracts take 1 unit of deposit at \( t = 0 \) and specify state contingent consumption \( d_j \) for \( t = 1 \) and \( c_j \) for \( t = 2 \). Let \( C \) denote the contract

\[
C = ((d_H, d_L), (c_H, c_L))
\]

where \( d_j \in \{d_H, d_L\} \) is the unit of date 1 good paid to \( \lambda_j \) fraction of early consumers who withdraw at \( t = 1 \), depending on the realization of liquidity shock \( j \). And \( c_j \in \{c_H, c_L\} \) is the unit of date 2 good paid to the remaining \((1 - \lambda_j)\) fraction of late consumers who withdraw at \( t = 2 \). Early consumers must withdraw at \( t = 1 \) in order to consume. Late consumers can either withdraw at \( t = 1 \) and consume \( d_j \) at \( t = 2 \) via private storage, or they can withdraw at \( t = 2 \) and receive \( c_j \) as consumption.

2.0.4 Asset Portfolio and Financial Opacity

If a bank has successfully attracted consumers, its bankers use consumer deposits plus equity capital to make investment at \( t = 0 \). The total resources available per contract to be invested is \((1 + e)\). The asset portfolio \( A \) of a bank can be expressed as

\[
A = (y, x, z)
\]

where \( y \) is the unit of good invested in the short asset, \( x \) is the unit of good invested in the risky asset, and \( z \) is the unit of good invested in the long asset. Therefore a fully-funded portfolio has the property \( x + y + z = 1 + e \).

At \( t = 0 \), the asset portfolio decision of any bank is not known to its consumers and the regulatory authority. This financial opacity can be thought of as a prohibitive cost for the regulatory authority and consumers to monitor the banks and it can be supported by two empirical observations. First, some financial institutions were able to window-dress balance sheets so that it was difficult for regulators to analyze the actual portfolio holdings. Second, regulators found it difficult to determine the precise return of exotic securities, many of which were often detected to be inefficient only after a crisis.

At \( t = 1 \), the asset portfolio becomes observable to consumers, who can determine if the bank is solvent, i.e. has sufficient resources to honor all contracts. The portfolio is assumed to be observable but not verifiable, therefore contract cannot be written on it. In other words, if banks have invested in the risky asset, they can restructure flows of resources in the \( G \) state so that there is no evidence of high profits from inefficient risk-taking. However, banks cannot come up with fake resources if

\(^{12}\) This incomplete consumption contract is similar to the deposit contract offered to consumers, or the debt contract issued to the creditors. Both deposit contract and debt contract are the mostly used contracts in reality rather than complex contracts. During the 2007-2009 financial crisis, creditors incurred most losses from the debt contracts.

\(^{13}\) According to FCIC (2011), Fannie Mae frequently adjusted asset categories so that the reported subprime holdings might be smaller than the actual subprime holdings.
they have incurred losses from the risky asset in the $B$ state. For example, if a community bank has lent money to subprime home owners. In the good times, subprime home owners do not default, depositors do not worry about the financial condition of the bank. Subprime lending may exist but it is difficult for depositors to find evidence. In the bad times, home owners default and depositors then know that subprime lending happened and start worrying about the financial condition of the bank.\footnote{The way assumed here is to account for the following fact. Prior to bankruptcy, some financial institutions made large profits from betting on risky assets and undertaking substantial downside risks. However, when seeing high profits in the good times, creditors usually did not claim those profits. For example, Long Term Capital Management obtained huge profits from risky convergence trades. There is an alternative way to model the asset substitution problem. It can be assumed that bankers can convert additional returns from risky asset into private benefits. In order words, banks can request subprime borrowers to pay prime interest rates plus private benefits. In that case, the balance sheets of risk-taking and non-risk-taking banks are the same in the $G$ state. It is not possible for consumers to find out based on the asset portfolio. The main result of the paper remains true under the alternative setup.}

\subsection*{2.0.5 Financial Market}

At $t = 1$ there is a market where banks can trade illiquid long and risky assets in exchange for liquidity, which is the date 1 good from short asset investment made at $t = 0$. When an illiquid asset is traded, the selling bank receives liquidity and transfers illiquid asset to the buying bank. The buying bank receives the illiquid asset and holds it to $t = 2$ to receive its return. Both long and risky assets can be traded in perfectly divisible units, and both illiquid assets are traded together indistinguishably for each unit of good deliverable at $t = 2$. Therefore market clearing requires that both illiquid assets must have the same date 1 price for 1 unit of good that it delivers at $t = 2$. Participation in this market is limited: banks can participate but individual consumers cannot.

Denote $q$ as the price of 1 unit of date 2 good in terms of date 1 good. In other words, $q$ is the price of 1 unit of illiquid asset in terms of liquidity. Therefore the price of 1 unit of risky asset is $qR_i$, for $i = G, B$ and the price of each entire long asset is $qF(z)$. The market-clearing price $q$ is determined by the amount of illiquid assets on sale and the amount of available liquidity. When a bank purchases illiquid assets, $\frac{1}{q}$ represents the gross return from $t = 1$ to $t = 2$. Illiquid assets can be traded with discount $q < 1$ or without discount $q = 1$. The price $q$ is bounded above by 1 since banks can always achieve zero net return by investing in short asset from $t = 1$ to $t = 2$.

At $t = 1$, in state $G$ there is an outside liquidation opportunity through which banks can sell one unit of date 2 good for one unit of date 1 good. This liquidation opportunity can be thought as the existence of abundant liquidity in good economic times that eases the liquidation of illiquid assets. It has been empirically documented that in good economic times there is abundant liquidity flown from real sector into financial sector and illiquid assets can usually be liquidated without discount.

\subsection*{2.0.6 Bank Payoff and Bankruptcy}

After paying all consumer contracts at $t = 1$ and $t = 2$, the residual resources of a bank will be paid to its shareholders (bankers) at $t = 2$. Within each bank, the number of bankers is the same as the number of consumers. Since all contracts are the same within a bank, the residual resources
of each contract is the same and will be paid to one banker. Therefore all bankers within a bank will receive the same residual payoff.

Bank run and bankruptcy can happen at $t = 1$. At $t = 1$, once portfolio becomes observable to consumers, if a bank does not have sufficient resources to honor all consumer contracts, bank run will happen. In a bank run, both early and late consumers will withdraw early at $t = 1$. Note that late consumers are consuming at $t = 2$ but they would withdraw early at $t = 1$. Late consumers would not wait till the next period since otherwise early consumers will withdraw and receive full amount of contracted payments. The consumers who wait will get even less.\(^{15}\)

The bank will then liquidate all illiquid asset holdings on the market. The liquidation proceeds plus any original holdings of liquidity will be paid out evenly to all consumers at $t = 1$. Bankers are of limited liability. If a bank does not have sufficient resources to honor contract to all consumers, the maximum resources consumers can claim are the total resources available at the bank. Each banker’s loss is up to the equity capital he provided at $t = 0$. In case of bankruptcy, the bankers will receive zero residual payoff.

### 2.0.7 Timing

The timing of the model is as follows. At $t = 0$, each bank offers a contract $C$ and each consumer chooses a bank to deposit his endowment. After collecting deposits from consumers, banks choose capital level $e$ and asset portfolio $A$. Any banker of banks without consumers will invest his endowment $w$ outside of the banking sector. At $t = 1$, aggregate state $i$ is first realized and bank portfolios become observable. Then liquidity shock $j$ is realized and subsequently consumer’s preference shock is realized.

\[(d_H, d_L) \quad (e_H, e_L)\]

\[t = 0 \quad t = 1 \quad t = 2\]

<table>
<thead>
<tr>
<th>Banks choose</th>
<th>Realizations of</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ contract</td>
<td>$i = G, B$ aggregate state</td>
</tr>
<tr>
<td>$e$ capital</td>
<td>$j = H, L$ liquidity shock</td>
</tr>
<tr>
<td>$A$ asset portfolio</td>
<td></td>
</tr>
</tbody>
</table>

### 2.1 The Constrained Efficient Allocation

In this section, I characterize the constrained efficient allocation by solving the social planner’s problem. The efficiency concept used here is the constrained efficiency. I let the planner offer consumption allocation in the same contract space as the banks. In this way, the allocation from planner’s problem can be used as a comparable benchmark to the allocations obtained in the banking equilibrium in section 3. Therefore the planner’s allocation here is constrained efficient.

\(^{15}\)Even if the initial losses can be small, no consumer wants to be the last one withdrawing as in Diamond and Dybvig (1983). Therefore bank run can potentially increase the losses at the bank because of costly liquidation.
The planner maximizes welfare of all consumers and the planner can utilize equity capital \( e \) by paying the usage cost \( K(e) \) to bankers.\(^{16}\) The planner always faces the same number of early consumers, since the total number of early consumers is fixed and equal to \((\frac{1}{2}\lambda_H + \frac{1}{2}\lambda_L)\). Therefore planner’s consumption plan does not need to depend on \( j \in \{H, L\} \). The planner’s consumption plan can be expressed as

\[
C = (d, c)
\]

where \( d \) is the consumption paid to the early consumers at \( t = 1 \) and \( c \) is the consumption paid to the late consumers at \( t = 2 \).

Because the risky asset is strictly dominated by the long asset, the social planner will not invest in the inefficient risky asset. The planner chooses a capital level \( e \) and invests in a portfolio of short and long assets. The planner’s asset portfolio is

\[
A = (y, 0, 1 + e - y)
\]

where \( y \) in the measure of total resources invested in the short asset and \((1 + e - y)\) is the total resources invested in the long asset. Note that the planner has a continuum of long asset investment opportunities to invest. So technically, the planner invests \((1 + e - y)\) units in each of the long assets in the interval \([0, 1]\), and will receive \( F(1 + e - y) \) from each of them at \( t = 2 \). The measure of total resources from long asset investment is \( F(1 + e - y) \times 1 = F(1 + e - y) \).\(^{17}\)

The planner maximizes welfare of all depositors by choosing consumption plan \( C = (d, c) \), capital level \( e \), and asset portfolio \( A = (y, 0, 1 + e - y) \), subject to state resources constraints.

\[
\max \quad E[U] = (\frac{1}{2}\lambda_H + \frac{1}{2}\lambda_L)u(d) + (1 - \frac{1}{2}\lambda_H - \frac{1}{2}\lambda_L)u(c)
\]

subject to

for \( i = G \)

\[
(\frac{1}{2}\lambda_H + \frac{1}{2}\lambda_L)d + (1 - \frac{1}{2}\lambda_H - \frac{1}{2}\lambda_L)c + K(e) = y + F(1 + e - y)
\]

(2.1)

for \( i = B \)

\[
(\frac{1}{2}\lambda_H + \frac{1}{2}\lambda_L)d \leq y
\]

(2.2)

\[
(\frac{1}{2}\lambda_H + \frac{1}{2}\lambda_L)d + (1 - \frac{1}{2}\lambda_H - \frac{1}{2}\lambda_L)c + K(e) = y + F(1 + e - y)
\]

(2.3)

In the objective function, \((\frac{1}{2}\lambda_H + \frac{1}{2}\lambda_L)\) is the total number of early consumers and \((1 - \frac{1}{2}\lambda_H - \frac{1}{2}\lambda_L)\) is the total number of late consumers. Constraints (2.1) and (2.3) are physical resources constraints across \( t = 1 \) and \( t = 2 \) in state \( G \) and state \( B \), respectively. In the \( B \) state, there is an additional constraint (2.2) which indicates that the planner must have sufficient date 1 good to pay all early

\(^{16}\)The planner will want to utilize some capital in the banking sector if the marginal cost of capital is lower than the marginal return of long asset investment.

\(^{17}\)The planner will spread investment evenly across the entire interval \([0, 1]\) of long assets because each long asset investment has diminishing marginal returns. It is not optimal to have two long assets with different levels of investment. Suppose the planner invests \( z_1 \) units in half of the long assets and \( z_2 \) units in the other half of the long assets, where \( z_1 \neq z_2 \). Then the planner can increase total return at \( t = 2 \) by making \( z_1 \) and \( z_2 \) closer.
consumers. There does not exist such constraint in the $G$ state since planner can always liquidate as many long assets as he wants using the liquidation opportunity.

Constraint (2.2) will be held in equality because it is not optimal for social planner to hold excess date 1 good (liquidity) from $t = 1$ to $t = 2$. Since $F'(z) > 1$ for all $z > 0$, the planner can always invest the excess liquidity in the long asset and receive strictly more resources at $t = 2$. Therefore the constraints (2.1), (2.2), and (2.3) effectively become

\[
\begin{align*}
\left(\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L\right)d &= y \\
(1 - \frac{1}{2} \lambda_H - \frac{1}{2} \lambda_L)c + K(e) &= F(1 + e - y)
\end{align*}
\]

(Returns from short asset investments are used to pay all early consumers at $t = 1$, and returns from long asset investments are used to pay all late consumers and the usage cost of capital at $t = 2$. Substitute $y$ from (2.4) into (2.5) to obtain a single constraint)

\[
F(1 + e - (\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L)d) = (1 - \frac{1}{2} \lambda_H - \frac{1}{2} \lambda_L)c + K(e)
\]

Using multiplier $\mu$ for constraint (2.6), the Lagrangian function is

\[
L = (\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L)u(d) + (1 - \frac{1}{2} \lambda_H - \frac{1}{2} \lambda_L)u(c) + \mu[F(1 + e - (\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L)d) - (1 - \frac{1}{2} \lambda_H - \frac{1}{2} \lambda_L)c - K(e)]
\]

Taking first order conditions with respect to $(d, c)$, the optimality conditions are

\[
\begin{align*}
u'(d) &= \mu F'(1 + e - y) \\
u'(c) &= \mu
\end{align*}
\]

Using (2.7) and (2.8), the optimal consumption plan is characterized by

\[
u'(d) = F'(1 + e - y)u'(c)
\]

To interpret this, suppose the planner reduces the consumption of one early consumer by $\epsilon$, and invests this $\epsilon$ resources in the long asset at $t = 0$. The additional investment return is $F'(1 + e - y)\epsilon$, which can be used to increase consumption to one late consumer at $t = 2$. The welfare of all consumers is not changed.

Taking first order condition with respect to $e$, the optimality condition for capital level is

\[
F'(1 + e - y) = K'(e)
\]

The marginal return of investment in long asset $F'(1 + e - y)$ should be equal to the marginal cost of capital $K'(e)$.

Denote $(d^*, c^*, e^*, y^*)$ as the optimal solution to the planner’s problem. The optimal solution $(d^*, c^*, e^*, y^*)$ can be solved using equations (2.4), (2.5), (2.9), and (2.10). It is a system of four equations in four variables. Once given specific functional forms of $u(c)$, $F(z)$, and $K(e)$, the optimal solution can be solved out explicitly.

In this constrained efficient allocation, social planner offers a smoothed consumption plan to maximize the welfare of all consumers. In particular, this constrained efficient allocation features
no investment in the inefficient risky asset. The planner chooses an optimal combination of short and long asset investments and utilizes the optimal level of equity capital to improve the utility of all consumers. This constrained efficient allocation will serve as a benchmark to the banking equilibrium allocations which will be studied in the next section.

3 Banking Equilibrium

In this section, I analyze the banking equilibrium in which banks offer contracts to consumers. In section 3.1, I analyze the banking equilibrium when financial opacity does not exist and risky asset cannot be invested. In section 3.2, when financial opacity is assumed to exist and risky asset can be invested, I show that the banking equilibrium generally involves inefficient risky asset investment. In section 3.3, I characterize the banking equilibrium with two types of banks by construction. The existence of the banking equilibrium with two types of banks is proved in section 3.4.

3.1 Banking Equilibrium without Financial Opacity

Suppose financial opacity does not exist and each bank’s asset portfolio is transparent at \( t = 0 \). In this case, investment in risky asset is detectable by consumers and can be prevented. All banks will choose only short asset and long asset in the asset portfolio. When consumers deposit their endowments with a bank, consumers can see the bank’s investment decision. Therefore consumers know that the true contract is the same as the promised contract.

Banks compete for consumers by offering the most attractive consumption contract. A representative bank chooses contract \( C = ((d_H, d_L), (c_H, c_L)) \) where \( d_j \in \{d_H, d_L\} \) is the consumption paid to early consumers at \( t = 1 \) and \( c_j \in \{c_H, c_L\} \) is the consumption paid to late consumers at \( t = 2 \).

After attracting consumers, banks raise equity capital \( e \) and choose the asset allocation between the short asset and long asset. A representative bank’s asset portfolio \( A \) is

\[
A = (y, 0, 1 + e - y)
\]

where \( y \) is the unit of good invested in the short asset and \( z = (1 + e - y) \) is the unit of good invested in the long asset. By assumption, banks invest \( x = 0 \) unit in the risky asset.

3.1.1 Payoffs of Banks

Since there are two aggregate states \( i \in \{G, B\} \) and two liquidity shocks \( j \in \{H, L\} \), there are four possible combinations of states \( ij \in \{GH, GL, BH, BL\} \). The date 1 price of one unit of date 2 good \( q \) can take two possible values \( q_i \in \{q_G, q_B\} \).

\(^{18}\)At \( t = 0 \), a representative bank chooses \((C, e, A)\). In each of the states \( i = G, B \), there is no uncertainty in aggregate liquidity demand and supply. The liquidity supply is the total amount of short assets invested by all banks at \( t = 0 \). The liquidity demand is the total amount of consumption good required to pay early consumers at \( t = 1 \).
Denote $V_{ij}$ as the state residual payoff per contract to be paid to the shareholder (banker) at $t = 2$. The state residual payoff $V_{ij}$ can take four possible values

$$V_{ij} \in \{V_{GH}, V_{GL}, V_{BH}, V_{BL}\}$$

Each state payoff $V_{ij}$ is in terms of date 2 good and it represents the amount of residual resources at the bank after paying all consumer contracts. Each state payoff $V_{ij}$ can be expressed as a function of contract $C$, capital level $e$, portfolio $A$, and prices $(q_G, q_B)$. $V_{ij}$ can take negative values when the amount of resources needed to pay all consumers is greater than the amount of resources available at the bank. In that case, the payoff to bankers is bounded below by zero because of limited liability.

When aggregate state is $G$, depending on the liquidity shock $j$, a bank faces state $GH$ or $GL$, the respective state residual payoff can be expressed as

$$V_{Gj} = F(1 + e - y) + \frac{y - \lambda_j d_j}{q_G} - (1 - \lambda_j)c_j$$

for $j = H, L$

A bank with liquidity shock $j$ pays $d_j$ to a fraction of $\lambda_j$ early consumers at $t = 1$. The bank will pay $c_j$ to the remaining $(1 - \lambda_j)$ late consumers at $t = 2$. The first term $F(1 + e - y)$ represents the total return in terms of date 2 good from long asset investment at $t = 0$.

In the second term, $(y - \lambda_j d_j)$ represents the net position of liquidity of this bank at $t = 1$. The bank has $y$ units of date 1 good (liquidity) and needs to pay $\lambda_j d_j$ units of date 1 good to the early consumers. If $(y - \lambda_j d_j) > 0$, this amount represents bank’s excess liquidity and the bank can use the excess liquidity to purchase long assets at the unit price $q_G$. At $t = 2$, the newly purchased long assets will deliver $\frac{y - \lambda_j d_j}{q_G}$ units of date 2 good. If $(y - \lambda_j d_j) < 0$, then the bank is short of liquidity and it needs to obtain liquidity by selling some long assets. Therefore at $t = 2$, the total amount of date 2 good will be reduced as $\frac{y - \lambda_j d_j}{q_G} < 0$. The gross return of purchasing illiquid asset at $t = 1$ and holding to $t = 2$ is $\frac{1}{q_G}$. The bank will purchase or sell illiquid long asset at $t = 1$ depending on the net position of liquidity.

When aggregate state is $B$, a bank faces state $BH$ or $BL$ and the respective state residual payoff is

$$V_{Bj} = F(1 + e - y) + \frac{y - \lambda_j d_j}{q_B} - (1 - \lambda_j)c_j$$

for $j = H, L$

In these two states, the price that the bank can purchase or sell one unit of illiquid asset is $q_B$.

The expected payoff of a bank to each banker can be computed using probability of each state. Given probabilities of aggregate state $Pr(i)$ and liquidity shock $Pr(j)$, the probability of each state $ij$ is $Pr(ij) = Pr(i)Pr(j)$. The expected payoff can be expressed as

$$E[V_{ij} | (C, e, A)] = \frac{1}{2}\pi(max[V_{GH}, 0] + max[V_{GL}, 0]) + \frac{1}{2}(1 - \pi)(max[V_{BH}, 0] + max[V_{BL}, 0])$$

The first part is the expected payoff in the $G$ state and the second part is the expected payoff in the $B$ state. In the following analysis, the expected payoff $E[V_{ij} | (C, e, A)]$ is sometimes written as $E[V_{ij}]$ for convenience.

Therefore for each aggregate state $i$, price $q$ will be the same regardless of realizations of idiosyncratic liquidity shocks.
The payoffs above are under the assumption that no late consumer withdraws early. In fact, if the contract pays weakly more resources at $t = 2$ than at $t = 1$ and there is no risk of bankruptcy, then no late consumer will withdraw early. This is the incentive compatibility condition for no early withdrawal and can be expressed as

$$d_H \leq c_H \quad \text{and} \quad d_L \leq c_L$$

As it will be shown later, since long asset generates positive net return, a utility maximizing consumption contract will always give a higher consumption at $t = 2$ than at $t = 1$. Therefore this condition will always hold for banks without a bank run.

### 3.1.2 Definition of Equilibrium without Financial Opacity

A banking equilibrium without financial opacity can be represented by the list of objects 

$$(C, e, A; q_G, q_B)$$

In equilibrium, the above objects must satisfy the following conditions

1. Optimal decisions of bankers: $(C, e, A)$ maximizes total expected payoff of bankers

   $$(C, e, A) \in \{(\hat{C}, \hat{e}, \hat{A}) \in \mathbb{R}_+^4 \times \mathbb{R}_+ \times \mathbb{R}_+^3 : E[V_{ij}((\hat{C}, \hat{e}, \hat{A})) + F_O(w - \hat{e}) \geq E[V_{ij}((\tilde{C}, \tilde{e}, \tilde{A})) + F_O(w - \tilde{e}), \forall (\tilde{C}, \tilde{e}, \tilde{A})]\}

2. Expected payoff of banks is equal to the opportunity cost of capital

   $$E[V_{ij}|(C, e, A)] = K(e)$$

3. Consumers choose the contract that gives the highest ex-ante expected utility

   $$C \in \{\hat{C} \in \mathbb{R}_+^4 : E[U(\hat{C})] \geq E[U(\tilde{C})], \forall \tilde{C}\}$$

4. Aggregate state resources constraints

   for $i = G$

   $$\frac{1}{2}(\lambda_H d_H + (1 - \lambda_H)c_H) + \frac{1}{2}(\lambda_L d_L + (1 - \lambda_L)c_L) + \frac{1}{2}(V_{GH} + V_{GL}) = F(z) + y$$

   for $i = B$

   $$\frac{1}{2}(\lambda_H d_H + \lambda_L d_L) \leq y$$

   $$\frac{1}{2}(\lambda_H d_H + (1 - \lambda_H)c_H) + \frac{1}{2}(\lambda_L d_L + (1 - \lambda_L)c_L) + \frac{1}{2}(V_{BH} + V_{BL}) = F(z) + y$$

Condition 1 states that given aggregate prices $(q_G, q_B)$, bankers choose the optimal $(C, e, A)$ to maximize the total expected payoff. The total expected payoff is the sum of expected payoff of
bank $E[V_{ij}[(\hat{C}, \hat{\epsilon}, \hat{A})]$ and payoff from the outside investment opportunity $F_O(w - \hat{\epsilon})$. There is no other decision vector $(\hat{C}, \hat{\epsilon}, \hat{A})$ that can result in higher total expected payoff.

Condition 2 indicates that the expected payoff of banks should be equal to the opportunity cost of capital. If the expected payoff of a bank $E[V_{ij}]$ is lower than its opportunity cost $K(e)$, then bankers of this bank can exit the banking sector and receive a higher total expected payoff $F_O(w)$. If the expected payoff of a bank $E[V_{ij}]$ is higher than its opportunity cost $K(e)$, then some other outside bankers have incentives to enter the banking sector. An entrant bank can steal consumers by offering a slightly more attractive contract and make higher total expected payoff to each banker than him staying outside and receiving $F_O(w)$. As a result, in equilibrium, all bankers will receive the same total expected payoff.

Condition 3 indicates that consumers choose the most attractive contract. Condition 4 states the aggregate resources constraints. Note that at $t = 1$ in the $G$ state banks can always liquidate long asset through the liquidation opportunity. In contrast, in the $B$ state the total available liquidity brought by the banks must be sufficient to cover the liquidity demand from all early consumers.

In equilibrium, bankers must choose $(C, e, A)$ to maximize the expected utility of the typical consumer, conditional on the participation of bankers. This is because under free entry, banks are competing with each other in offering the most attractive contract to consumers. To ensure no entry or exit of bankers, the expected payoff of banks $E[V_{ij}]$ must be equal to the opportunity cost of capital $K(e)$. The equilibrium conditions 1, 2, and 3 are equivalent to the following utility maximization condition: given prices $(q_G, q_B)$, bankers choose $(C, e, A)$ to maximize expected utility of consumer

$$(C, e, A) \in \{(\hat{C}, \hat{\epsilon}, \hat{A}) \in \mathbb{R}_+^4 \times \mathbb{R}_+ \times \mathbb{R}_+^3 : E[U(\hat{C})] \geq E[U(\hat{C})] \text{ such that } E[V_{ij}[(\hat{C}, \hat{\epsilon}, \hat{A})] = K(\hat{\epsilon})]\}

Intuitively, competition forces banks not only to receive payoff that is exactly the same as cost of capital, but also to offer contract in the utility maximizing way. Suppose there is a bank whose expected payoff is equal to the cost of capital and it offers a non-utility maximizing contract. Then another entrant bank can steal consumers by offering a contract with higher expected utility and with slightly less resources. A entrant bank will do so since it can obtain positive payoff in addition to the cost of capital.

3.1.3 Optimization Problem

Taking prices of illiquid asset $(q_G, q_B)$ as given, bankers choose contract $C = (d_H, d_L, c_H, c_L)$, capital level $e$, and asset portfolio $A = (y, 0, 1 + e - y)$ to maximize expected utility of typical consumer. The optimization problem is subject to four types of constraints: participation constraint, resources constraints, incentive constraints, and capital constraint.

$$\max E[U] = \frac{1}{2} [\lambda_H u(d_H) + (1 - \lambda_H) u(c_H)] + \frac{1}{2} [\lambda_L u(d_L) + (1 - \lambda_L) u(c_L)]$$
subject to
\[ E[V_{ij}|(C,e,A)] = K(e) \quad (3.1) \]
\[ V_{ij} \geq 0, \quad \forall ij \in \{GH,GL,BH,BL\} \quad (3.2) \]
\[ d_H \leq c_H, \quad d_L \leq c_L \quad (3.3) \]
\[ e \geq \bar{e} \quad (3.4) \]

The first constraint (3.1) is the participation constraint for bankers. The second constraints (3.2) are the resources constraints since banks cannot have negative resources in any of the four possible states. The third constraints (3.3) are the incentive compatibility constraints for no early withdrawal of late consumers. The fourth constraint (3.4) is the capital requirement constraint.

Use \( \mu \) as multiplier on constraint (3.1), \( \mu_{ij} \) as multipliers on the four constraints in (3.2), \( \mu_j \) as multipliers on the two constraints in (3.3), and \( \omega \) as multiplier on constraint (3.4). The Lagrangian function is
\[
L = \frac{1}{2} \lambda_H u(d_H) + (1 - \lambda_H)u(c_H) + \frac{1}{2} \lambda_L u(d_L) + (1 - \lambda_L)u(c_L) \\
+ \mu [\frac{1}{2} \pi (V_{GH} + V_{GL}) + \frac{1}{2} (1 - \pi)(V_{BH} + V_{BL}) - K(e)] \\
+ \mu_{GH} V_{GH} + \mu_{GL} V_{GL} + \mu_{BH} V_{BH} + \mu_{BL} V_{BL} \\
+ \mu_H (c_H - d_H) + \mu_L (c_L - d_L) + \omega (e - \bar{e})
\]

Take the first order conditions with respect to \( (d_H, d_L) \) and \( (c_H, c_L) \). The analysis focuses on the situation when none of the resources constraints binds.\(^\text{19}\) The optimality conditions for consumptions are
\[
u'(d_H) = u'(d_L) = \mu [\pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] \\
u'(c_H) = u'(c_L) = \mu
\]

Therefore the optimal consumption contract is characterized by
\[ d_H = d_L \quad \text{and} \quad c_H = c_L \quad (3.5) \]
\[ u'(d_j) = [\pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] u'(c_j) \quad \text{for} \quad j = H, L \quad (3.6) \]

The optimality conditions indicate that consumption within each time period \( t = 1, 2 \) should be equalized, regardless of the realization of liquidity shock \( j \). The consumption across two time periods is governed by the gross interest rate \( [\pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] \) between \( t = 1 \) and \( t = 2 \). The gross interest rate is the expected marginal return of short asset investment. By investing in 1 unit of short asset at \( t = 0 \) and use this unit of liquidity to purchase illiquid assets at \( t = 1 \), banks can obtain \( [\pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] \) as the expected return of date 2 good at \( t = 2 \).

Taking the first order condition with respect to \( y \), the optimal portfolio decision is

\(^{19}\)When some of the resources constraints is binding, the optimal solution is also well characterized and the details are shown in appendix.
\[ F'(1 + e - y) = \pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B} \]  

(3.7)

The marginal return of investment in long asset is equal to the expected marginal return of investment in short asset. Consider the last \( \epsilon > 0 \) units of resources that a single bank wants to invest at \( t = 0 \). If the bank invests in long asset, it will receive \( F'(1 + e - y) \epsilon \) at \( t = 2 \). If the bank invests in short asset, then at \( t = 1 \) it can use \( \epsilon \) units of liquidity to purchase long asset and on expectation the bank will receive \( [\pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] \epsilon \) units of good at \( t = 2 \). In this case, the bank is indifferent between investments in the short asset and in the long asset at \( t = 0 \).

According to (3.5), denote \( d = d_j \) and \( c = c_j \), for \( j = H, L \). Together with (3.6) and (3.7), the optimal consumption contract is characterized by

\[ u'(d) = F'(1 + e - y)u'(c) \]  

(3.8)

Take the first order condition with respect to \( e \). When capital requirement constraint is not binding, the optimality condition for capital level is

\[ F'(1 + e - y) = K'(e) \]  

(3.9)

The marginal return of long asset investment is equal to the marginal cost of equity capital. If \( F'(1 + e - y) > K'(e) \), banks can utilize a higher level of capital to generate more resources and offer a more attractive contract. If \( F'(1 + e - y) < K'(e) \), then the investment return from the last unit of capital is not sufficient to cover the cost of using it. Additional resources have to be taken away from consumers to cover the cost of capital, and therefore it is not optimal.

### 3.1.4 Market Clearing

In order for markets to clear, the prices \((q_G, q_B)\) need to satisfy two conditions. First, \((q_G, q_B)\) must clear the markets when aggregate state is \( G \) and \( B \) at \( t = 1 \). Secondly, \((q_G, q_B)\) must be the case that all banks are choosing the optimal portfolio decision. This means for the last unit of resources invested, the marginal return of long asset investment should be equal to the marginal expected return of short asset investment.

At \( t = 1 \), since the aggregate measure of banks is \( 1 \) and each bank holds \( y \) units of liquidity, aggregate liquidity supply is \( y \times 1 = y \). In both \( G \) state and \( B \) state, the aggregate liquidity demand is fixed to be \( \frac{1}{2} \lambda_H d_H + \frac{1}{2} \lambda_L d_L \), regardless of the realizations of bank-specific liquidity shocks. An illiquid asset is liquidated without discount if \( q_i = 1 \): the amount of date 2 good the illiquid asset is going to deliver at \( t = 2 \) is transformed into the same amount of date 1 good.

In the \( G \) state, since the outside liquidation opportunity can always liquidate 1 unit of date 2 good into 1 unit of date 1 good, the illiquid assets must be liquidated without discount in the market, i.e. \( q_G = 1 \). This is because if a bank needs to liquidate long asset, it can always do so by

\(^{20}\)The indifference of investment holds on the last unit of resources. Since banks are price takers, from each bank’s perspective, the marginal return of long asset investment is diminishing and the marginal expected return of short asset investment in constant. Mechanically, a bank continues to invest in the long asset until the marginal return of long asset investment is equal to the marginal expected return of short asset investment. The banks then invest all remaining resources in the short asset. In doing so, the total investment return is maximized.
using the liquidation opportunity, and therefore it will never sell in the market if $q_G < 1$.

In the $B$ state, if there is excess liquidity: aggregate liquidity supply is greater than aggregate liquidity demand, then illiquid assets must be liquidated without discount

$$y > \frac{1}{2} \lambda_H d_H + \frac{1}{2} \lambda_L d_L \quad \Rightarrow \quad q_B = 1$$

Assume for contradiction that there is excess liquidity and $q_B < 1$, then the financial market in the $B$ state cannot clear. This is because some banks will have the opportunity of buying illiquid long asset and make a positive net return

$$\frac{1}{q_B} - 1 > 0 \quad \text{from} \quad t = 1 \quad \text{to} \quad t = 2.$$ Some other banks will not have the trading opportunity and will hold liquidity from $t = 1$ to $t = 2$, thus obtaining 0 rate of net return. Only when $q_B = 1$, two groups of banks are willing to hold both long asset and excess liquidity between $t = 1$ and $t = 2$, both yielding 0 rate of net return.

However, the prices $(q_G = 1, q_B = 1)$ cannot be equilibrium, since the long asset would dominate the short asset between $t = 0$ and $t = 2$. The return of long asset is greater than the expected return of short asset for all $y > 0$

$$F'(1 + e - y) > \pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B} = \frac{\pi}{1} + \frac{1 - \pi}{1} = 1$$

The intuition is that long asset would always be liquidated without discount and thus short asset always generates 0 rate of net return. Therefore no bank is willing to invest in short asset at $t = 0$. This is contradictory to bank’s portfolio $A = (y, 0, z)$ with $y > 0$. Therefore in equilibrium it is not possible to have excess liquidity in the $B$ state.

In equilibrium, the prices of illiquid asset $(q_G, q_B)$ must ensure that banks are choosing the optimal levels of short asset and long asset between $t = 0$ and $t = 2$. As a result, there must be exact liquidity at $t = 1$ in the $B$ state

$$\frac{1}{2} \lambda_H d_H + \frac{1}{2} \lambda_L d_L = y \quad (3.10)$$

Using $q_G = 1$ together with equation (3.7), price $q_B$ should satisfy

$$F'(1 + e - y) = \pi \frac{1}{1} + (1 - \pi) \frac{1}{q_B} \quad (3.11)$$

In this case, choosing short asset and long asset is indifferent for banks at $t = 0$. It is worthwhile to briefly discuss the return of liquidity investment and aggregate prices $(q_G, q_B)$. Although investment in liquidity (short asset) generates 0 net return from $t$ to $t + 1$ for $t = 0, 1$, the liquidity holdings can potentially generate strictly positive returns from purchasing illiquid assets at $t = 1$. The positive returns provide incentives for banks to invest in the liquidity at $t = 0$. Intuitively, long asset investment is productive and liquidity should be costly to acquire. Banks obtain benefits from holding liquidity, and the magnitude of this benefit will adjust to match the forgone gains from long asset investment. In equilibrium, the prices $(q_G, q_B)$ must adjust such that banks receive the same marginal return from liquidity investment and long asset investment.

The optimal decisions $(d^*, e^*, e^*, y^*)$ and equilibrium price $q_B$ can be solved out using equations (3.1), (3.8), (3.9), (3.10), and (3.11). It is a system of five equations in five variables.

In this economy, banks insure each other against the idiosyncratic liquidity shocks through the
financial market. At $t = 1$, half of the banks face high liquidity demand $\lambda_H$ and they need to liquidate some of the long assets to pay early consumers. The other half of the banks face low liquidity demand $\lambda_L$ and they will use the excess liquidity to purchase long assets. The optimal consumption contract is independent of idiosyncratic liquidity shock. Banks choose the optimal combination of short asset and long asset to maximize the utility of consumers. Banks also utilize the optimal level of capital to generate resources and use capital as buffer to help smooth consumption for consumers against aggregate and idiosyncratic shocks. The expected payoff of banks paid to each banker is the same as the opportunity cost of capital $K(e)$. The banking equilibrium is risk-free as it features no inefficient risky asset investment and bankruptcy.

3.1.5 Constrained Efficiency of Equilibrium without Financial Opacity

The allocation under the risk-free banking equilibrium can be compared to the planner’s constrained efficient allocation. When financial opacity does not exist, risky asset cannot be invested and the source of inefficiency is eliminated. As a result, the banking equilibrium allocation can potentially be constrained efficient.

**Proposition 1.** When financial opacity does not exist and the state resources constraints (3.2) and capital constraint (3.4) do not bind, banking equilibrium allocation is constrained efficient.

**Proof.** See appendix.

The proof follows by showing that the optimality conditions of social planner’s problem and those of the banking equilibrium give the same allocation, when constraints (3.2) and (3.4) do not bind. Intuitively, banks compete with each other in offering the most attractive contract to consumers. Banks utilize the optimal level of capital and choose the optimal portfolio to maximize resources and structure contract in the way that consumption smoothing across states is best achieved. The efficiency of banking equilibrium should carry the same intuition as the first welfare theorem that competitive equilibrium generates efficiency.

3.2 Banking Equilibrium with Financial Opacity

In this section, I analyze the banking equilibrium when financial opacity exists and banks are able to invest in the risky asset. Without financial opacity, the banking equilibrium does not involve inefficient risk-taking and its allocation can be constrained efficient. In the existence of financial opacity, the banking equilibrium is considerably different.

3.2.1 Existence of Inefficient Risk-Taking

When financial opacity exists, the constrained efficient allocation cannot be supported. The risk-free banking equilibrium in section 3.1 fails to exist because the portfolio decision $A^*$ is no longer optimal for all banks. Some banks can invest in the risky asset and obtain a strictly higher expected
payoff. The incentive to invest in the risky asset comes with the possibility of bankruptcy in the 
B state, which will lead to low level of consumption to a fraction of consumers.

To see this in detail, consider the risk-free banking equilibrium in section 3.1 with optimal 
contract $C^*$, capital level $e^*$, and asset portfolio $A^*$, where $A^*$ is given by

$$A^* = (y^*, 0, 1 + e^* - y^*)$$

The portfolio $A^*$ is no longer the optimal decision for all banks. Suppose a bank indexed by 
$m$ offers the same prevailing contract $C^*$ as other banks. After attracting consumers, this bank $m$ 
can change its asset portfolio to the following

$$A^m = (0, x^m, 0)$$

where $x^m = 1 + e^*$

Bank $m$ replaces all short asset and long asset investments by investments in the risky asset. If 
aggregate state is $G$, bank $m$ receives high payoff $R_G$ from each risky asset and can obtain additional 
residual payoffs by paying the prevailing contract $C^*$. When aggregate state is $B$, consumers of 
bank $m$ run the bank at $t = 1$ due to insufficient resources to honor all contracts. The bank is 
bankrupt and bankers receive 0 as residual payoff.

The expected payoff of the risk-taking bank $m$ is

$$E[V_{ij}^m|(C^*, e^*, A^m)] = \pi(\frac{1}{2}V_{GH}^m + \frac{1}{2}V_{GL}^m)$$

Bank $m$ still faces four possible states $ij$, but its bankers only receive payoffs in state $Gj \in 
\{GH, GL\}$. The state payoff $V_{Gj}^m$ is given by

$$V_{Gj}^m = (1 + e^*)R_G - \frac{\lambda_j d_j}{q_G} - (1 - \lambda_j)e_j$$

for $j = H, L$

where $(1 + e^*)R_G$ is the total return from risky asset investment and bank $m$ sells illiquid risky 
asset at $t = 1$ to obtain liquidity to pay early consumers.\(^{21}\)

The risk-taking bank $m$ can obtain a higher expected payoff if the following condition is satisfied

$$E[V_{ij}^m|(C^*, e^*, A^m)] > E[V_{ij}^m|(C^*, e^*, A^*)] = K(e^*) \quad (3.12)$$

where $E[V_{ij}^m|(C^*, e^*, A^*)]$ is the expected payoff of other banks in the risk-free equilibrium $(C^*, e^*, A^*)$.

Note that in this case bank $m$’s capital level $e^m$ is assumed to be the same as other bank’s 
optimal capital level $e^*$. This is true if the capital requirement is binding $e^* = \bar{e}$. If the capital 
requirement is not binding $e^* > \bar{e}$, then bank $m$ can potentially choose a lower capital level at $\bar{e}$ 
and obtain an even higher payoff

$$E[V_{ij}^m|(C^*, \bar{e}, A^m)] > E[V_{ij}^m|(C^*, e^*, A^m)] \quad \text{for } e^* > \bar{e}$$

A risk-taking bank obtains the highest expected payoff when it chooses the lowest possible level 
of capital.\(^{22}\) In this case, risk-taking bank will choose capital level to be the same as the capital 
requirement $e^m = \bar{e}$.

\(^{21}\)There is always sufficient liquidity in the $G$ state because of the outside liquidation opportunity. 

\(^{22}\)This result is proved in section 3.3. As risky asset is inefficient $E[R] < 1$, the marginal return of risky asset 
investment is not sufficient to cover the cost of capital $K'(e) > 1$. By reducing the level of capital, the net change in 
risk-taking bank’s expected payoff is positive.
Given any $\bar{e} > 0$ set by authority, denote set $T(\bar{e})$ as the set of all risky assets with return $R_G$ that will lead to bank risk-taking and destruction of the risk-free equilibrium

$$T(\bar{e}) = \{ R_G \in \mathbb{R}_+: E[V_{ij}^n|(C^*, \bar{e}, A^n)] > E[V_{ij}|(C^*, e^*, A^*)] = K(e^*) \}$$

(3.13)

This is a requirement on the return structure of the risky asset to break the risk-free equilibrium. Inefficient risk-taking exists if the risky asset return $R_G$ satisfies condition (3.13). Specific functional forms of $u(c)$, $F(z)$, and $K(e)$ are needed to generate explicit condition on $R_G$. Intuitively, even risk-taking banks only receive payoffs in the $G$ state, as long as the risky asset generates a sufficiently attractive return in the $G$ state, condition (3.13) will be satisfied.

Therefore for any risky asset with $R_G \in T(\bar{e})$, the risk-free equilibrium fails to exist under financial opacity. If the risky asset generates an attractive return in the $G$ state, some banks would have incentives to pursue higher payoffs from risky asset investment.\textsuperscript{23} In this case, risky asset investment by some banks cannot be avoided in equilibrium.

**Proposition 2.** In the existence of financial opacity, for any $\bar{e} > 0$, if the risky asset satisfies $R_G \in T(\bar{e})$, then the risk-free equilibrium $(C^*, e^*, A^*; q_G, q_B)$ cannot exist. Inefficient risk-taking exists under financial opacity.

The risk-free equilibrium fails to exist because financial opacity makes investment in the risky asset feasible. Even the risky asset is strictly dominated by other safe assets from society’s perspective, leveraged banks may have a different perspective and find it optimal to invest. This inconsistence in views is larger the larger the spread of risky asset $R_G$. The inefficient risk-taking results in a market failure and welfare loss to some consumers. Financial opacity, leverage, limited liability, and risky asset jointly contribute to the inefficiency.

**Remark:** In the banking equilibrium with financial opacity, if a bank chooses an asset portfolio $A = (y, x, z)$ with positive units of risky asset $x > 0$, then this risk-taking bank’s contract cannot reflect risk-taking and this bank must fail in the $B$ state.

The key problem in this model is that consumers are not sure whether a bank has invested in the risky asset. Since investment in the risky asset is strictly dominated by long asset and short asset, any bank investing in the risky asset will be receiving a strictly lower amount of resources in expectation than non risk-taking banks. Offering a honest contract that reflects risky asset investment will not attract any consumer. Suppose a bank invests in positive amount of risky assets $x > 0$ and it offers a contract that it never defaults. Then the most attractive non-default contract offered by this bank must be giving strictly lower expected utility than contracts offered by banks without risky asset investment. Therefore any non-default contract offered by the risk-taking bank will not be able to attract consumers.

\textsuperscript{23}This potential risk-taking activity does not pertain to a specific bank. Any bank in the interval $[0, 1]$ has the option to invest in the risky asset. In this case it is a measure zero bank that performs risk-taking. While the social cost of such risky asset investment is zero, the equilibrium is still destroyed since the portfolio decision $A^*$ is no longer optimal for all banks. Profitable risk-taking by a positive measure of banks can be easily constructed as well.
In order to attract consumers, banks with risky asset investment must offer a contract from which consumers do not identify risky asset investment. In other words, based on promised contract, the risk-taking banks need to be perceived as non risk-taking banks. As a result, in the $B$ state risk-taking banks will not have sufficient resources to honor all contracts. Therefore these banks will face bank runs and become bankrupt in the $B$ state.

It is worth to briefly discuss several properties of the model and conjecture what type of equilibrium could possibly exist. In this model, the major source of inefficiency is the risky asset. Without financial opacity, banking equilibrium features no inefficient risk-taking and competition among banks forces banks to offer the most attractive contract to consumers. Banking equilibrium allocation can be constrained efficient.

The setup of financial opacity is novel and realistic. The existence of financial opacity allows for potential inefficient risk-taking by banks, and yet undetected by the consumers. The example on the existence of inefficient risk-taking gives a rationale of how the banking equilibrium could possibly be under financial opacity. Since consumers cannot see the portfolio when selecting the contract, risk-taking banks can possibly survive in the equilibrium by offering the same contract as other banks that do not invest in the risky asset. In the good times, risk-taking banks could obtain additional payoffs from the risky asset. In the bad times, these banks become bankrupt and the loss to each banker is bounded by his initial capital contribution because of limited liability. Consumers cannot identify risk-taking banks when selecting a contract and could therefore incur low consumption when they happen to be with a risk-taking bank in the bad times.

### 3.3 Equilibrium with Two Types of Banks

In this section, I show that with financial opacity, there exists a banking equilibrium with two types of banks: safe banks and risky banks. A bank is defined to be a safe bank if it does not invest in the risky asset $x = 0$. A bank is defined to be a risky bank if it invests positive units in the risky asset $x > 0$. The equilibrium is characterized by construction and its existence is proved.

I first make assumptions on the behaviors of safe bankers and risky bankers. Later it can be shown that in equilibrium the assumed behaviors are indeed optimal for both safe and risky bankers. Safe bankers are assumed to maximize expected utility of consumers and they are always able to honor the contract at all states. The risky bankers are assumed to offer the same contract as safe bankers but they maximize their own total expected payoff. When selecting contracts, consumers cannot distinguish between two types of banks. In the $G$ state, risky banks obtain additional payoffs from risky asset investment and pay the safe bank’s contract. In the $B$ state, risky banks become bankrupt and safe banks will purchase illiquid assets from failing risky banks. Safe bank’s contract includes information about the potential bankruptcies and asset liquidations of risky banks.

Let $n \in \{S, R\}$ denote the type of banks, where $S$ denotes for safe banks and $R$ denotes for risky banks. I look at the optimization problem of a representative bank in each bank type. The equilibrium is a symmetric equilibrium: banks of the same type choose the same decisions. Both types of banks offer the same prevailing consumption contract.
There are two possible values for price $q \in \{q_G, q_B\}$, since the price $q_i$ does not depend on the idiosyncratic liquidity shock or distribution of shocks. In the $G$ state, no bank fails and the aggregate liquidity demand and supply are fixed. In the $B$ state, the law of large numbers applies in each of the risky bank group and safe bank group. The relative frequency of liquidity shocks is the same in each group as in the population. So half of the safe/risky banks will face high liquidity shock $H$ and half of the safe/risky banks will face low liquidity shock $L$.

### 3.3.1 Payoffs of Safe Banks

Let $C^S$ denote the true contract of safe banks

$$C^S = ((d^S_H, d^S_L), (c^S_H, c^S_L))$$

Safe bank’s promised contract is the same as its true contract.

Let $e^S$ denote the capital level of safe banks. The asset portfolio of safe banks $A^S$ at $t = 0$ is

$$A^S = (y^S, 0, 1 + e^S - y^S)$$

By assumption safe banks do not invest in the risky asset, therefore $x^S = 0$. Safe banks invest $y^S$ units of good in the short asset and $z^S = (1 + e^S - y^S)$ units in the long asset.

The set of state residual payoffs of safe banks is

$$V^S_{ij} \in \{V^S_{GH}, V^S_{GL}, V^S_{BH}, V^S_{BL}\}$$

where each $V^S_{ij}$ can be expressed as a function of contract $C^S$, capital level $e^S$, portfolio $A^S$, and prices of illiquid asset ($q_G, q_B$).

When aggregate state is $G$, depending on the state $GH$ or $GL$, the state residual payoff is

$$V^S_{Gj} = F(1 + e^S - y^S) + \frac{y^S - \lambda_j d^S_j}{q_G} - (1 - \lambda_j)c^S_j$$

for $j = H, L$

Note that when the net position of liquidity $(y^S - \lambda_j d^S_j) > 0$, safe banks will purchase both long asset and risky asset indistinguishably as illiquid asset from the market at $t = 1$. When the net position of liquidity $(y^S - \lambda_j d^S_j) < 0$, safe banks sell long asset to obtain liquidity.

When aggregate state is $B$, depending on the state $BH$ or $BL$, the state residual payoff is

$$V^S_{Bj} = F(1 + e^S - y^S) + \frac{y^S - \lambda_j d^S_j}{q_B} - (1 - \lambda_j)c^S_j$$

for $j = H, L$

The expected payoff of safe banks can be computed by taking expectations

$$E[V^S_{ij}((C^S, e^S, A^S)) = \frac{1}{2}\pi(\max[V^S_{GH}, 0] + \max[V^S_{GL}, 0]) + \frac{1}{2}(1 - \pi)(\max[V^S_{BH}, 0] + \max[V^S_{BL}, 0])$$

### 3.3.2 Payoffs of Risky Banks

Let $C^R$ denote the true contract of risky banks in the $B$ state

$$C^R = ((d^R_H, d^R_L), (c^R_H, c^R_L))$$

The terms represent the actual consumption paid to consumers in the $B$ state. In the $G$ state, the true contract of risky banks is the same as true contract of safe banks. The promised contract
that risky banks offer is the same as safe banks $C^S$.

By assumption, risky banks invest in positive amount of risky assets. Let $e^R$ denote the capital level chosen by the risky banks. The asset portfolio $A^R$ of risky banks at $t = 0$ can be expressed as

$$A^R = (y^R, x^R, 1 + e^R - x^R - y^R)$$

such that $x^R > 0$

where $y^R$ is the units of good invested in the short asset, $x^R$ is the units invested the risky asset, and $z^R = (1 + e^R - x^R - y^R)$ is the units invested in the long asset.

Since risky banks offer the same promised contract $C^S$ as safe banks, given $C^S$, the state payoffs of risky banks can be expressed as functions of safe bank’s contract $C^S$, capital level $e^R$, portfolio $A^R$, and prices $(q_G, q_B)$. The set of state residual payoffs of risky banks is

$$V_{ij}^R \in \{V_{GH}^R, V_{GL}^R, V_{BH}^R, V_{BL}^R\}$$

Intuitively these state residual payoffs are different from the state residual payoffs of safe banks. But keep in mind that both types of banks offer the same promised contract $C^S$.

When aggregate state is $G$, depending on liquidity shock $H$ or $L$, the state residual payoff is

$$V_{Gj}^R = F(1 + e^R - x^R - y^R) + x^R R_G - \frac{y^R - \lambda_j d_j^S}{q_G} - (1 - \lambda_j) c_j^S$$

for $j = H, L$

Risky banks pay $d_j^S$ to each early consumer and $c_j^S$ to each late consumer. Depending on the net position of liquidity $(y^R - \lambda_j d_j^S)$, at $t = 1$ risky banks either buy illiquid asset (long or risky) or sell risky asset at unit price $q_G$.

When aggregate state is $B$, since risky asset will generate a low return $R_B$ at $t = 2$, risky banks do not have sufficient resources to honor all consumer contracts. Therefore the state residual payoffs will be negative

$$V_{Bj}^R < 0$$

for $j = H, L$

At $t = 1$ consumers of risky banks will observe that their banks are lack of resources to honor all contracts, and all consumers will withdraw early at $t = 1$. In this case, risky banks are bankrupt and they will liquidate the entire long and risky asset holdings. The liquidation proceeds plus original holdings of liquidity will be divided equally among all consumers (pro rata). The true contract of risky banks in times of default is

$$d_H^R = d_L^R = \left[F(1 + e^R - x^R - y^R) + x^R R_B\right]q_B + y^R$$

$$c_H^R = c_L^R = \left[F(1 + e^R - x^R - y^R) + x^R R_B\right]q_B + y^R$$

where $F(1 + e^R - x^R - y^R)$ is the amount of date 2 good deliverable from long asset at $t = 2$. And $x^R R_B$ is the amount of date 2 good deliverable from risky asset at $t = 2$. The sum of the two amounts multiplied by $q_B$ gives the amount of date 1 good as liquidation proceeds.

Note that when there is a bankruptcy, the idiosyncratic liquidity shock $j$ does not matter. Both early and late consumers will be paid at $t = 1$ and all consumers will receive the same liquidation payoff at $t = 1$. Note that consumers will not receive $d_j^S$ even if $d_j^S < d^R$ since all consumers can claim that they are late consumers. 

$^{24}$
consumers can save the liquidation payoff via private storage at 0 rate of net return and consume at \( t = 2 \). For simplicity, denote the consumption under bankruptcy as \( d^R = d^*_j = c^*_j \), for \( j = H, L \).

The expected payoff of risky banks is
\[
E[V^R_{ij}((C^S, e^R, A^R)]] = \frac{1}{2}\pi(\max[V^R_{BH}, 0] + \max[V^R_{BL}, 0])
\]
Bankers of risky banks only receive payoffs in the \( G \) state. Because of bankruptcy in the \( B \) state, the state residual payoffs \( V^R_{BH} \) and \( V^R_{BL} \) are negative. Under limited liability, the actual payoff to risky bankers is 0 in states \( BH \) and \( BL \).

### 3.3.3 Definition of Equilibrium with Two Types of Banks

A banking equilibrium with two types of banks consists of the list of objects
\[
(\theta, \theta^E; C^S, C^R, e^S, e^R, A^S, A^R; q_G, q_B)
\]

1. \( \theta \in [0, 1] \) is the fraction of risky banks in the economy. \( \theta^E \in [0, 1] \) is consumer’s belief about the fraction of risky banks in the economy.

2. Consumption contract \( C^n \in \{C^S, C^R\} \): \( C^S = (d^S_H, d^S_L, c^S_H, c^S_L) \) is the true contract offered by safe banks and \( C^R = (d^R_H, d^R_L, c^R_H, c^R_L) \) is the true contract offered by risky banks in the \( B \) state.

3. Capital level \( e^n \in \{e^S, e^R\} \): \( e^S \) is the capital level of safe banks and \( e^R \) is the capital level of risky banks.

4. Asset portfolio \( A^n \in \{A^S, A^R\} \): portfolio of safe banks is \( A^S = (y^S, x^S, z^S) \) with \( x^S = 0 \) and \( y^S + z^S = 1 + e^S \). Portfolio of risky banks is \( A^R = (y^R, x^R, z^R) \) with \( x^R > 0 \) and \( y^R + x^R + z^R = 1 + e^R \).

5. Price of illiquid asset \( q_i \in \{q_G, q_B\} \): \( q_G \) is the price in state \( G \) and \( q_B \) is the price in state \( B \).

In equilibrium, the above objects must satisfy the following conditions

1. Optimal decisions of safe bankers: \((C^S, e^S, A^S)\) maximizes total expected payoff
   \[
   (C^S, e^S, A^S) \in \{(\tilde{C}^S, \tilde{e}^S, \tilde{A}^S) \in \mathbb{R}_+^3 \times \mathbb{R}_+ \times \mathbb{R}_+^3 \} : \\
   E[V^S_{ij}((\tilde{C}^S, \tilde{e}^S, \tilde{A}^S)] + F_O(w - \tilde{e}^S) \geq E[V^S_{ij}([\tilde{C}^S, \tilde{e}^S, \tilde{A}^S)] + F_O(w - \tilde{e}^S), \forall (\tilde{C}^S, \tilde{e}^S, \tilde{A}^S)]
   \]

2. Optimal decisions of risky bankers: \((C^S, e^R, A^R)\) maximizes total expected payoff
   \[
   (C^S, e^R, A^R) \in \{(\tilde{C}^R, \tilde{e}^R, \tilde{A}^R) \in \mathbb{R}_+^3 \times \mathbb{R}_+ \times \mathbb{R}_+^3 \} : \\
   E[V^R_{ij}([\tilde{C}^R, \tilde{e}^R, \tilde{A}^R)] + F_O(w - \tilde{e}^R) \geq E[V^R_{ij}([\tilde{C}^R, \tilde{e}^R, \tilde{A}^R)] + F_O(w - \tilde{e}^R), \forall (\tilde{C}^R, \tilde{e}^R, \tilde{A}^R)]
   \]
3. Expected payoff of each type of bank is equal to its opportunity cost of capital

\[ E[V_j^S(C_j^S, e_j^S, A_j^S)] = K(e_j^S), \quad \text{if } \theta \in [0, 1) \]

\[ E[V_j^R(C_j^R, e_j^R, A_j^R)] = K(e_j^R), \quad \text{if } \theta \in (0, 1] \]

4. Consumers choose the utility maximizing contract based on belief \( \theta^E \).

5. Consistency of consumer’s belief: \( \theta^E = \theta \).

6. Consumers choose to participate in the banking sector

\[ E[U(C_i^S, \theta^E)] \geq U^{Aut} = U(C^{Aut}, 0) \]

where \( C^{Aut} = (d^{Aut}, d^{Aut}, c^{Aut}, c^{Aut}) \) and \( d^{Aut} = c^{Aut} = 1 \)

7. Aggregate state resources constraints

for \( i = G \)

\[
\frac{1}{2}(\lambda_H d_H^S + (1 - \lambda_H)d_H^R) + \frac{1}{2}(\lambda_L d_L^S + (1 - \lambda_L)d_L^R) \\
+ \theta \frac{1}{2}(\max[V_{GH}^R, 0] + \max[V_{GL}^R, 0]) + (1 - \theta) \frac{1}{2}(\max[V_{GH}^S, 0] + \max[V_{GL}^S, 0]) \\
= \theta[F(z^R) + x^R R_G + y^R] + (1 - \theta)[F(z^S) + y^S]
\]

for \( i = B \)

\[
(1 - \theta) \frac{1}{2}(\lambda_H d_H^S + \lambda_L d_L^S) + \theta \frac{1}{2}(\lambda_H d_H^R + (1 - \lambda_H)c_H^R) + \frac{1}{2}(\lambda_L d_L^R + (1 - \lambda_L)c_L^R) \leq \theta y^R + (1 - \theta) y^S \\
(1 - \theta) \left[ \frac{1}{2}(\lambda_H d_H^S + (1 - \lambda_H)c_H^S) + \frac{1}{2}(\lambda_L d_L^S + (1 - \lambda_L)c_L^S) \right] \\
+ \theta \left[ \frac{1}{2}(\lambda_H d_H^R + (1 - \lambda_H)c_H^R) + \frac{1}{2}(\lambda_L d_L^R + (1 - \lambda_L)c_L^R) \right] \\
+ (1 - \theta) \frac{1}{2}(\max[V_{BH}^S, 0] + \max[V_{BL}^S, 0]) \\
= \theta[F(z^R) + x^R R_B + y^R] + (1 - \theta)[F(z^S) + y^S]
\]

8. Asset market clearing condition: for \( i = B, q_B \) satisfies

\[
(1 - \theta) y^S - (1 - \theta) \left( \frac{1}{2} \lambda_H d_H^S + \frac{1}{2} \lambda_L d_L^S \right) = \theta [F(z^R) + x^R R_B] q_B
\]

Conditions 1 and 2 state that safe bankers and risky bankers choose the optimal \((C^n, e^n, A^n)\) to maximize their respective total expected payoff. The total expected payoff is the sum of expected payoff of bank \( E[V_j^S(C_j^S, e_j^S, A_j^S)] \) and the payoff from the outside investment opportunity \( F_G(w - e^n) \). Taking aggregate prices \((q_G, q_B)\) as given, there is no other decision vector \((C^n, e^n, A^n)\) that can result in higher total expected payoff. Condition 2 indicates that choosing safe bank’s contract \( C^S \) should be the optimal decision for risky banks to maximize total expected payoff.
Condition 3 indicates that the expected payoff of banks $E[V_{ij}^n|(C^n, e^n, A^n)]$ must be equal to the opportunity cost of capital $K(e^n)$ for both types of banks. Therefore the total expected payoff is the same and equal to $F_O(w)$ for both types of bankers. Neither safe bankers nor risky bankers can obtain a higher total expected payoff by changing to the other type.

Conditions 4 and 5 indicate that consumers choose the utility maximizing contract based on their belief $\theta^E$ and this belief must be consistent with the true fraction of risky banks $\theta$ in equilibrium. Condition 6 indicates that in order for consumers to be willing to deposit their endowments with the banks, the expected utility of consumers has to be greater than that of autarky: consuming the endowment via private storage.

Condition 7 states the aggregate resources constraints. At $t = 1$ in the $B$ state, the total date 1 good (liquidity) paid to early consumers of safe banks and all consumers of risky banks must be less than or equal to the total date 1 good available in the economy. There is no such constraint in the $G$ state at $t = 1$ because of the outside liquidation opportunity. For $i = G, B$, the total resources paid to consumers and bankers across $t = 1$ and $t = 2$ should be equal to the total resources generated from all asset investments. Condition 8 states the market clearing condition at $t = 1$. In the $B$ state, the price of illiquid asset $q_B$ should be such that all the excess liquidity of safe banks is used to purchase all the illiquid assets on sale by the risky banks.

### 3.3.4 Solving for Equilibrium with Two Types of Banks

The banking equilibrium with two types of banks can be characterized by construction in two steps. First, an equilibrium candidate is proposed and this equilibrium candidate satisfies a subset of the equilibrium conditions. Second, one additional condition is identified to ensure the equilibrium candidate satisfies all equilibrium conditions.

A feasible allocation can be specified by the vector $(\theta, A^R, e^R)$. Given any measure of risky banks $\theta \in [0, 1]$ and the portfolio of risky banks $A^R = (y^R, x^R, z^R)$, the aggregate level of risky asset can be determined and is equal to $\theta x^R$. The aggregate level of liquidity held by risky banks is $\theta y^R$. Given the aggregate levels of risky asset and liquidity held by risky banks, safe bank’s utility maximization problem can be solved to obtain the optimal contract $C^{S*}(\theta, A^R, e^R)$, which is a function of $(\theta, A^R, e^R)$. Then the optimal contract $C^{S*}(\theta, A^R, e^R)$ can be used together with risky bank’s portfolio $A^R$ and capital level $e^R$ to compute the expected payoff of risky banks $E[V_{ij}^R|(C^{S*}(\theta, A^R, e^R), e^R, A^R)]$.

An equilibrium candidate is a feasible allocation that satisfies certain equilibrium conditions by construction. An equilibrium candidate can be obtained using the following algorithm.

1. Specify a vector $(\theta, A^R, e^R)$, where $\theta \in [0, 1]$ and $e^R \in [\bar{e}, w]$.

---

25If $E[V_{ij}^n|(A^n, e^n, C^n)] > K(e^n)$, there exist potential entries by outside bankers. If $E[V_{ij}^n|(A^n, e^n, C^n)] < K(e^n)$, type $n$ bankers will exit the banking sector and receive $F_O(w)$.

26Consumers understand that there is a fraction of risky banks in the economy and thus form expectation about the fraction of risky banks. Consumers also understand the actual consumption they will receive in the $B$ state if they are with a risky bank.
2. Solve the utility maximization problem of safe banks to obtain safe bank’s optimal contract 
\( C_{S}^{\ast}(\theta, A^{R}, e^{R}) \), sometimes denoted as \( C^{S\ast} \) for short.

3. Compute the expected payoff of risky banks \( E[V_{ij}^{R}|(C^{S\ast}(\theta, A^{R}, e^{R}), e^{R}, A^{R})] \) under the assumption that risky banks offer the same contract \( C^{S\ast} \) as safe banks.

4. An equilibrium candidate is obtained if
\[
E[V_{ij}^{R}|(C^{S\ast}(\theta, A^{R}, e^{R}), e^{R}, A^{R})] = K(e^{R}).
\]
The key step is to search for a vector \((\theta, A^{R}, e^{R})\) that satisfies \( E[V_{ij}^{R}|(C^{S\ast}, e^{R}, A^{R})] = K(e^{R}) \). In this case, the expected payoff of risky banks is equal to the opportunity cost of capital. For the safe banks, the counterpart of this payoff condition \( E[V_{ij}^{S}|(C^{S\ast}, e^{S\ast}, A^{S\ast})] = K(e^{S\ast}) \) will always hold as a result of optimization in step 2. These two payoff conditions ensure no-entry and no-exit of each type of banks. Basically bankers of both risky and safe banks receive the same total expected payoff and neither type has incentive to change to the other type or exit. Also outside bankers have the same total expected payoff and they do not have incentive to enter the banking sector.

### 3.3.5 Optimal Decisions of Risky Banks

The entire set of feasible allocations can be specified by the set of all possible vectors \( \{(\theta, A^{R}, e^{R})\} \). However, it is only necessary to consider a subset of feasible allocations because most feasible allocations do not satisfy certain equilibrium conditions. Here I derive the optimality conditions for risky bank’s portfolio \( A^{R} \) and capital level \( e^{R} \). These optimality conditions substantially reduce the set of feasible allocations that need to be considered for an equilibrium candidate.

For a feasible allocation to be supported in equilibrium, risky bankers do not have incentive to change portfolio \( A^{R} \) or capital level \( e^{R} \). Given asset prices \( (q_{G}, q_{B}) \), risky bankers choose asset portfolio and capital level that give them the highest total expected payoff.

**Lemma 1.** Consider any feasible allocation specified by \((\theta, A^{R}, e^{R})\), where \( A^{R} = (y^{R}, x^{R}, z^{R}) \) and \( y^{R} + x^{R} + z^{R} = 1 + e^{R} \). If \( R_{G} > \frac{1}{q_{G}} \), then this feasible allocation cannot be supported in equilibrium if \( y^{R} > 0 \) or \( z^{R} > 0 \).

**Proof.** See appendix.

The intuition is straightforward. Since bankers of risky banks only receive payoffs in the \( G \) state, from risky bank’s perspective, investment in risky asset is dominant comparing to short asset and long asset. In the \( G \) state, the price of illiquid asset \( q_{G} = 1 \) because of the existence of outside liquidation opportunity. As a result, investment in the risky asset generates a higher expected return than in the short asset since \( R_{G} > \frac{1}{q_{G}} = 1 \). Because of the easiness of obtaining liquidity in the \( G \) state, risky banks would want to replace all short assets by higher-yielding risky assets. Also since \( R_{G} > F'(z), \forall z \), marginal investment in the risky asset is yielding a higher return than the long asset. So risky banks do not choose any long asset.

**Lemma 2.** Given any capital requirement \( \bar{e} > 0 \), in equilibrium the optimal level of capital chosen by the risky banks is the same as the capital requirement \( e^{R\ast} = \bar{e} \).
Proof. See appendix. ■

Since risky asset is inefficient $E[R] < 1$ and using capital requires paying positive net return $K'(e) > 1$, the marginal return of using an additional unit of capital is not sufficient to cover the opportunity cost of this unit of capital. Additional resources have to be taken away from risky bankers to pay for the usage of capital. Therefore risky bankers can obtain the highest total expected payoff by choosing the lowest possible capital level.

Using Lemma 1 and Lemma 2, in equilibrium risky banks must choose the lowest level of capital $e^{R*} = \bar{e}$ that meets the capital requirement and they invest the entire portfolio in the risky asset. Therefore the optimal portfolio of risky banks

$$A^{R*} = (0, x^{R*} = 1 + \bar{e}, 0)$$

Any feasible allocation $(\theta, A^{R}, e^{R})$ with $e^{R} \neq \bar{e}$ or $x^{R} \neq 1 + \bar{e}$ cannot be equilibrium and it is not necessary to consider such allocation for an equilibrium candidate. For any $\theta \in [0, 1]$, the vector $(\theta, A^{R*}, e^{R*}) = (0, 1 + \bar{e}, 0)$ can be represented by $(\theta, A^{R*}, e^{R*})$. Therefore when searching for an equilibrium candidate, a feasible allocation can be found by simply changing $\theta$.

### 3.3.6 Optimization Problem

Given the fraction of risky banks $\theta \in [0, 1]$, taking prices of illiquid asset $(q_G, q_B)$ as given, safe banks choose contract $C^S$, capital level $e^S$, and asset portfolio $A^S$ to maximize expected utility of typical consumer with expectation $\theta$. The expected utility of consumer depends on the true contracts of safe and risky banks. The optimization problem is subject to four types of constraints: participation constraint, resources constraints, incentive constraints, and capital constraint.

$$\max \ E[U] = \frac{1}{2} \pi [\lambda_H u(d^S_H) + (1 - \lambda_H)u(c^S_H)] + \frac{1}{2} \pi [\lambda_L u(d^S_L) + (1 - \lambda_L)u(c^S_L)]$$

$$+ (1 - \pi)\frac{1}{2} (1 - \theta) [\lambda_H u(d^S_H) + (1 - \lambda_H)u(c^S_H)]$$

$$+ (1 - \pi)\frac{1}{2} (1 - \theta) [\lambda_L u(d^S_L) + (1 - \lambda_L)u(c^S_L)]$$

$$+ \theta (1 - \pi) u(d^R)$$

subject to

$$E[V^S_i | (C^S, e^S, A^S)] = K(e^S) \quad (3.21)$$

$$V^S_i \geq 0, \ \forall \ ij \in \{GH, GL, BH, BL\} \quad (3.22)$$

$$d^S_H \leq c^S_H, \ \ d^S_L \leq c^S_L \quad (3.23)$$

$$e^S \geq \bar{e} \quad (3.24)$$

In the objective function, the first line represents the expected utility when aggregate state is $G$. The second line and third line represent the expected utility when aggregate state is $B$ and consumer is with a safe bank. The forth line represents the expected utility when aggregate state is $B$ and consumer is with a failing risky bank. The first constraint (3.21) is the participation constraint that ensures bankers receive exactly the same payoff from banks as the opportunity cost of capital. The second constraints (3.22) are the state resources constraints. The third constraints (3.23) are
the incentive constraints for no early withdrawal of late consumers and the fourth constraint (3.24) is the capital requirement constraint.

Use multiplier $\mu$ on constraint (3.21), $\mu_{ij}$ on the four constraints in (3.22), $\mu_j$ on constraints (3.23), and $\omega$ on constraint (3.24). The Lagrangian function is

$$\mathcal{L} = \frac{1}{2} \pi [\lambda_H u(d_H^S) + (1 - \lambda_H)u(c_j^S)] + \frac{1}{2} \pi [\lambda_L u(d_L^S) + (1 - \lambda_L)u(c_L^S)]$$

$$+ (1 - \theta)(\frac{1}{2}(1 - \pi)[\lambda_H u(d_H^S) + (1 - \lambda_H)u(c_H^S)]$$

$$+ (1 - \theta)(\frac{1}{2}(1 - \pi)[\lambda_L u(d_L^S) + (1 - \lambda_L)u(c_L^S)]$$

$$+ \theta(1 - \pi)u(d^R)$$

$$+ \mu_1(\frac{1}{2}\pi(V_{GH}^S + V_{GL}^S) + \frac{1}{2}(1 - \pi)(V_{BH}^S + V_{BL}^S) - K(e^S)]$$

$$+ \mu_GH V_{GH}^S + \mu_GL V_{GL}^S + \mu_BH V_{BH}^S + \mu_BL V_{BL}^S$$

$$+ \mu_H(c_H^S - d_H^S) + \mu_L(c_L^S - d_L^S) + \omega(e^S - \bar{c})$$

Take the first order conditions with respect to $(d_H^S, d_L^S)$ and $(c_H^S, c_L^S)$. The analysis focuses on the situation when none of the state resources constraints is binding. When some of the resources constraints binds, the main results of the paper are unaffected and the details are shown in appendix.

The optimality conditions for consumption are

$$u'(d_H^S) = u'(d_L^S) = \mu_{\frac{1}{2}}[\frac{1}{2}\pi + (1 - \pi)\frac{1}{2}(1 - \pi)]$$

$$u'(c_H^S) = u'(c_L^S) = \mu_{\frac{1}{2}}[\frac{1}{2}\pi + (1 - \theta)\frac{1}{2}(1 - \pi)]$$

Therefore the optimal consumption contract is characterized by

$$d_H^S = d_L^S \quad \text{and} \quad c_H^S = c_L^S \quad (3.25)$$

$$u'(d^S) = [\frac{1}{q_G} + (1 - \pi)\frac{1}{q_B}]u'(c_j^S) \quad \text{for} \quad j = H, L \quad (3.26)$$

Similar to the case without financial opacity, the optimality conditions indicate that consumption within each time period $t = 1, 2$ should be equalized, regardless of liquidity shock $j$. The consumption across two time periods is governed by the gross interest rate $(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B})$ between $t = 1$ and $t = 2$.

Taking the first order condition with respect to $y^S$, the optimal portfolio decision is

$$F'(1 + e^S - y^S) = \frac{\pi}{q_G} + (1 - \pi)\frac{1}{q_B} \quad (3.27)$$

The marginal return of long asset investment is equal to the expected marginal return of short asset investment. At $t = 0$, safe banks are indifferent between the last unit investment in the short asset and in the long asset.

According to (3.25), denote $d^S_j = d_j^S$ and $c^S = c_j^S$, for $j = H, L$. Together with (3.26) and (3.27), the optimal consumption contract is characterized by

$$u'(d^S) = F'(1 + e^S - y^S)u'(c^S) \quad (3.28)$$
Take the first order condition with respect to $e$. When the capital requirement does not bind, the optimality condition for capital level is

$$F'(1 + eS - yS) = K'(eS)$$

(3.29)

At the optimal level of capital, the marginal return from long asset investment is equal to the marginal cost of equity capital.

### 3.3.7 Market Clearing

The market clearing prices $(q_G, q_B)$ need to satisfy two conditions. First, $(q_G, q_B)$ must clear the market in each of the $G$ and $B$ states at $t = 1$. Second, and importantly, $(q_G, q_B)$ must ensure that both types of banks are choosing the optimal portfolio decisions. So safe banks are indifferent between the last unit of investment in short asset and long asset at $t = 0$. And risky banks strictly prefer risky asset than long asset and short asset.

In the $G$ state, illiquid assets must be liquidated without discount $q_G = 1$ in the market, since banks can always sell 1 unit of date 2 good into 1 unit of date 1 good through the outside liquidation opportunity. Using $q_G = 1$ together with equation (3.27), price $q_B$ should satisfy

$$F'(1 + eS - yS) = \frac{\theta}{1 + \bar{e}} R_B q_B$$

(3.30)

In the $B$ state, market clearing requires that $q_B$ should satisfy

$$(1 - \theta)yS - (1 - \theta)(\frac{1}{2}\lambda_H d_H^S + \frac{1}{2}\lambda_L d_L^S) = \theta(1 + \bar{e})R_B q_B$$

(3.31)

On the left-hand-side, the first term $(1 - \theta)yS$ is the total liquidity available at $t = 1$. The second term $(1 - \theta)(\frac{1}{2}\lambda_H d_H^S + \frac{1}{2}\lambda_L d_L^S)$ is the total liquidity required to pay the early consumers of safe banks. Therefore the left-hand-side is the amount of excess liquidity that will be used to purchase risky assets. On the right-hand-side, $\theta(1 + \bar{e})R_B$ is the total units of date 2 good that will be delivered by all risky assets.

The optimal decisions $(d^*, e^*, e^r, y^r)$ and equilibrium price $q_B$ can be solved using equations (3.21), (3.28), (3.29), (3.30), and (3.31). It is a system of five equations in five variables. When some of the constraints (resources or capital) is binding, the problem can be solved as a regular Kuhn-Tucker problem and is shown in appendix.

Using the optimal contract of safe banks $C^{S*}(\theta, A^{R*}, e^{R*})$ with the optimal capital level $e^{R*}$ and portfolio $A^{R*}$ of risky banks, the expected payoff of risky banks $E[V^R_{ij} | (C^{S*}(\theta, A^{R*}, e^{R*}), e^{R*}, A^{R*})]$ can be computed. Then depending on the value of $E[V^R_{ij}]$, adjust $\theta \in [0, 1]$ to search for an equilibrium candidate.

### 3.4 Existence of Equilibrium with Two Types of Banks

In this section I prove the existence of the banking equilibrium with two types of banks. To do so, I first show the existence of an equilibrium candidate. There exists a fraction of risky banks $\theta \in (0, 1)$ such that $E[V^R_{ij} | (C^{S*}(\theta, A^{R*}, e^{R*}), e^{R*}, A^{R*})] = K(e^{R*})$. For the feasible allocation under this $\theta$,
two types of bankers receive the same total expected payoff. Therefore neither type of bankers has incentive to change to the other type. Then I show that the equilibrium candidate satisfies all other equilibrium conditions with some additional condition.

Using results from Lemma 1 and Lemma 2, in equilibrium risky banks must choose \( e^{Rs} = \bar e \) and \( x^{Rs} = 1 + \bar e \). It is only necessary to consider the set of feasible allocations specified by \( \theta \in [0,1] \). By construction, given any \( \theta \), safe banks compete against each other in offering the utility maximizing contract. Therefore the expected payoff of safe banks will always be equal to the opportunity cost of capital \( E[V^{S}_ij](C^{Ss}(\theta, A^{Rs}, e^{Rs}), e^{Ss}, A^{Ss})] = K(e^{Ss}) \).

### 3.4.1 Payoff of Risky Banks \( E[V^{R}_{ij}] \) and \( \theta \)

I analyze the mapping from the fraction of risky banks \( \theta \) to the expected payoff of risky banks \( E[V^{R}_{ij}](C^{Ss}(\theta, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})] \), or \( E[V^{R}_{ij}] \) for short. Given \( \theta \), the expected payoff \( E[V^{R}_{ij}] \) is computed using the algorithm for equilibrium candidate. As \( \theta \) changes, the optimal contract of safe banks \( C^{Ss}(\theta, A^{Rs}, e^{Rs}) \) changes. As a result, \( E[V^{R}_{ij}](C^{Ss}(\theta, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})] \) changes as well. When \( \theta \) is small, the expected payoff of risky banks \( E[V^{R}_{ij}] \) is higher than opportunity cost of capital. When \( \theta \) is large, the expected payoff of risky banks \( E[V^{R}_{ij}] < K(e^{Rs}) \).

**Lemma 3.** There exists a \( \theta_1 \in [0,1] \) such that \( E[V^{R}_{ij}](C^{Ss}(\theta_1, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})] > K(e^{Rs}) \): the expected payoff of risky banks is higher than opportunity cost of capital. There exists a \( \theta_2 \in [0,1] \) such that \( E[V^{R}_{ij}](C^{Ss}(\theta_2, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})] < K(e^{Rs}) \): the expected payoff of risky banks is lower than opportunity cost of capital.

**Proof.** See appendix.

The existence of \( \theta_1 \) has been established in section 3.2 when showing that risk-free equilibrium cannot exist because of profitable inefficient risk-taking. For any \( \bar e > 0 \), if risky asset satisfies \( R_G \in T(\bar e) \), then the expected payoff of risky banks is higher than opportunity cost of capital. Intuitively, when \( \theta \) is small and close to 0, there is not much assets on sale from risky banks in the \( B \) state. Therefore the optimal contract of safe banks cannot be very attractive, which makes it inexpensive for risky banks to mimic. This cannot be equilibrium since more safe banks would want to become risky banks.

The existence of \( \theta_2 \) is intuitive. When \( \theta \) is large and close to 1, there is a large number of risky banks failing in the \( B \) state. As a result, there is a large amount of illiquid assets on sale in the \( B \) state and the price of illiquid asset \( q_B \) will be very low. Safe banks will benefit from the deeply discounted asset price and are able to offer a very attractive contract \( C^{Ss} \). Therefore risky banks will receive a very low expected payoff \( E[V^{R}_{ij}] \) after mimicking the highly costly contract in the \( G \) state. This cannot be equilibrium since more risky banks would want to become safe banks or exit.

### 3.4.2 Continuity of \( E[V^{R}_{ij}] \) in \( \theta \)

For any \( \theta \in [0,1] \), the optimal contract of safe banks \( C^{Ss}(\theta, A^{Rs}, e^{Rs}) \) can be obtained and the expected payoff of risky banks \( E[V^{R}_{ij}] \) can be computed.
Lemma 4. Given any $\theta \in [0,1)$, apply the algorithm to compute the expected payoff of risky banks $E[V_{ij}^R | (C^{S*}(\theta, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})]$. The expected payoff $E[V_{ij}^R | (C^{S*}(\theta, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})]$ is continuous in $\theta$.

Proof. See appendix.

The expected payoff of risky banks $E[V_{ij}^R]$ is a continuous function of optimal contract of safe banks $C^{S*}$, optimal decisions of risky banks $e^{Rs}$ and $A^{Rs}$, and aggregate prices $(q_G, q_B)$. Therefore it is to show that all the input variables in $E[V_{ij}^R]$ are continuous functions of $\theta$.

The essence of the proof is to use maximum theorem to show that the optimal contract terms $(d_{ij}^{S*}, c_{ij}^{S*})$ are continuous functions of $\theta$. Given any prices $(q_G, q_B)$, the contract terms $(d_{ij}^{S*}, c_{ij}^{S*})$ are optimal solutions of safe bank’s utility optimization problem. After verifying that the assumptions of the theorem are satisfied, the maximum theorem can be applied to prove the continuity of $(d_{ij}^{S*}, c_{ij}^{S*})$ in prices $(q_G, q_B)$. Finally, the market clearing condition (3.31) is used to generate the continuity of $(q_G, q_B)$ in $\theta$.

Proposition 3. There exists a $\theta^* \in (0,1)$ such that $E[V_{ij}^R | (C^{S*}(\theta^*, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})] = K(e^{Rs})$: the expected payoff of risky banks is equal to the opportunity cost of capital.

Proof. Intuitively, there exists an endogenously determined fraction of risky banks $\theta^*$ such that both types of bankers received the same total expected payoff. By Lemma 3, there exists a $\theta_1 \in [0,1)$ such that $E[V_{ij}^R | (C^{S*}(\theta_1, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})] > K(e^{Rs})$. Also there exists a $\theta_2 \in [0,1)$ such that $E[V_{ij}^R | (C^{S*}(\theta_2, A^{Rs}, e^{Rs}), e^{Rs}, A^{Rs})] < K(e^{Rs})$. By Lemma 4, $E[V_{ij}^R] - K(e^{Rs})$ is a continuous function of $\theta$. Therefore there exists a $\theta^* \in (\theta_1, \theta_2)$ such that the expected payoff of risky banks is equal to the opportunity cost of capital. The intuition is illustrated in the graph below.

![Graph showing $E[V_{ij}^R]$ and $K(e^{Rs})$]

It has been established that there exists an equilibrium candidate at $\theta^*$. There is one additional condition that needs to be imposed to ensure that the equilibrium candidate under $\theta^*$ is a banking equilibrium with two types of banks.

Condition 1 Given any $\bar{e} > 0$, there exists an equilibrium candidate under $\theta^*(\bar{e})$. Consumers are willing to participate in the banking sector if $E[U(C^{S*}(\theta^*(\bar{e}), A^{Rs}, e^{Rs}), \theta^*(\bar{e})] \geq E[U(C^{Aut}, 0)]$. 36
The banking sector can exist only if consumers are willing to deposit their endowments with banks. The expected utility obtained from a bank contract must be weakly higher than the expected utility of autarky. The expected utility depends on the true contracts of safe banks and risky banks. Intuitively, if capital requirement $\bar{e}$ is low and the fraction of risky banks $\theta^*(\bar{e})$ is large, the possibility of being with a risky bank is high. Therefore consumers may not join the banking sector. Generally a lower bound on the capital requirement is needed to ensure consumer participation.

**Proposition 4.** For any $\bar{e} > 0$ and the equilibrium candidate $\theta^*(\bar{e}) \in (0, 1)$. If condition 1 is satisfied, then there exists a banking equilibrium with two types of banks at $\theta^*(\bar{e})$.

**Proof.** See appendix.

The proof is to show that the equilibrium candidate under $\theta^*(\bar{e})$ satisfies all the equilibrium conditions. Essentially, the assumed decisions of bankers should indeed be maximizing their total expected payoff, respectively. The first part is to show that changing contract cannot lead to higher payoffs. Offering a less attractive contract loses consumers, while offering a more attractive contract makes bankers a lower payoff. It is assumed that consumers form the same belief about the fraction of risky banks over all contracts offered and consumers choose the utility maximizing contract under the belief. Therefore offering any non utility maximizing contract with twisted consumptions across two periods will not be accepted by consumers.

The second part is to show that changing capital level $e$ and portfolio $A$ cannot lead to higher payoffs. By construction, the total expected payoff is the same for both safe bankers and risky bankers. Therefore no banker wants to change to the other type. One interesting possibility is that a bank may invest all resources in the short asset. The bank aims to profit from the $B$ state and becomes bankrupt in the $G$ state. This possibility, together with other possibilities, cannot lead to a higher total expected payoff with details shown in appendix.

## 4 Policy Implications

In this section, I study the impact of capital requirements on the banking equilibrium and welfare of consumers. In section 4.1, I study the impact of capital requirements on consumer welfare when financial opacity does not exist. In section 4.2, when financial opacity is assumed to exist, I study the impact of capital requirements on risk-taking and welfare.

The Basel Accord is considered as the most important policy that governs the risk-taking behavior of banks. After the global financial crisis in 2007-2009, international regulators have decided to update the Basel II to Basel III, which imposes a higher capital requirement ratio. This consensus among regulators across the world is aiming to reduce leverage and curb inefficient risk-taking. The idea behind this is that by forcing a larger stake contributed from shareholders, banks will be more cautious in making investment decisions. Using the model developed in this paper, I can assess the policy implications of capital requirements on risk-taking and welfare.
4.1 Capital Requirement without Financial Opacity

Suppose financial opacity does not exist and the banking authority imposes a capital requirement $\bar{e} > 0$. It is shown in Proposition 1 that banks choose the socially optimal level of capital $e^* = e^{**}$ when the state resources constraints and capital constraint do not bind. When some of the resources constraints binds, banks choose a slightly higher level of capital $e^* > e^{**}$ to help smooth consumption across states.

Denote $e^*_0$ as the optimal level of capital chosen by banks without any capital requirement. For any $\bar{e} \leq e^*_0$, the capital requirement constraint is not binding and has no impact on the banking equilibrium. For clarity of notations, denote $e^*(\bar{e})$ as the optimal level of capital chosen by banks under the capital requirement $\bar{e}$. Similarly, $(d^*(\bar{e}), c^*(\bar{e}), y^*(\bar{e}))$ represent the optimal contract and portfolio decisions under $\bar{e}$. Therefore $e^*(\bar{e}) = e^*_0$ for $\bar{e} \leq e^*_0$ and $e^*(\bar{e}) > e^*_0$ for $\bar{e} > e^*_0$.

Suppose $\bar{e} > e^*_0$. Constraint on capital level is strictly binding and the optimal level capital level is equal to the capital requirement $e^*(\bar{e}) = \bar{e}$. The optimality condition for capital level becomes

$$F'(1 + e - y) + \omega = K'(e)$$

Since the capital constraint holds with equality, the multiplier $\omega > 0$. Using the optimality condition for short asset $y$

$$F'(1 + e^*(\bar{e}) - y^*(\bar{e})) = \pi \frac{1}{qG} + (1 - \pi) \frac{1}{qB} < K'(e^*(\bar{e}))$$

In this case, the marginal return of investment in the short and long assets is not sufficient to cover the the marginal cost of capital. Banks are forced to utilize additional units of capital, whose cost cannot be fully financed in the banking sector.

**Lemma 5.** Consider any strictly binding capital requirement $\bar{e} > e^*_0$, where $e^*_0$ is the optimal level of capital chosen by banks without any capital requirement. If $\bar{e}$ increases, the investment in long asset increases $\frac{\partial z^*(\bar{e})}{\bar{e}} > 0$, the marginal return of long asset investment decreases $\frac{\partial F'(z^*(\bar{e}))}{\bar{e}} < 0$, and the consumption for late consumers decreases $\frac{\partial c^*(\bar{e})}{\bar{e}} < 0$.

**Proof.** See appendix.

When banks are forced to operate with a higher level of capital, banks will invest part of the increased capital into the long asset $F(z)$. It is not possible to invest all of the increased capital in the short asset as the consumption smoothing condition will be violated. Since the long asset has diminishing marginal returns, as $\bar{e}$ increases, the marginal return $F'(z^*(\bar{e}))$ decreases. This means the expected marginal return of short asset investment decreases as well. Late period consumption $c^*(\bar{e})$ decreases because the gross interest rate between $t = 1$ and $t = 2$ decreases and additional resources are needed to pay the cost of capital.

**Proposition 5.** When financial opacity does not exist, consider any strictly binding capital requirement $\bar{e} > e^*_0$. If $\bar{e}$ increases, the welfare of consumers decreases.

**Proof.** See appendix.
Under a strictly binding capital requirement, banks are forced to use a higher level of capital than they want. Since marginal cost of capital $K'(\bar{e})$ is increasing and higher than marginal return $F'(z^*(\bar{e}))$, the marginal cost of additional capital cannot be fully financed within the banking sector. Banks have to take some resources away from consumers to pay for the usage of capital. Therefore increasing a binding capital requirement strictly reduces welfare, and the welfare loss is larger the larger the increase.\footnote{This is true even if the marginal cost of capital $K'(e)$ is constant and greater than marginal return $F'(z^*(\bar{e}))$. The marginal welfare loss is increasing for the same amount of reduction in consumption. The decrease in welfare is even larger when the marginal cost of capital $K'(e)$ is increasing.}

Without financial opacity, imposing or increasing a capital requirement can only reduce welfare of consumers. There is no market failure and capital requirement is not needed.

4.2 Capital Requirement with Financial Opacity

When financial opacity exists, inefficient risk-taking generally exists in equilibrium. There is a role of capital requirement in reducing risk-taking and improving welfare. Given a capital requirement $\bar{e}_1 > 0$, there is a banking equilibrium at $\theta^*(\bar{e}_1)$. Suppose authority raises the capital requirement to $\bar{e}_2 > \bar{e}_1$. Since risky banks always choose the lowest level of capital, risky banks are forced to use a higher level of capital $e^{R*} = \bar{e}_2$. As risky asset is inefficient $E[R] < 1 < K'(e)$, the marginal return of risky asset investment cannot cover the marginal cost of capital. Therefore if the fraction of risky banks remains unchanged at $\theta^*(\bar{e}_1)$, expected payoff of risky banks $E[V_{ij}^{R}]$ will be lower than cost of capital $K(e^{R*} = \bar{e}_2)$.

**Proposition 6.** Given any capital requirements $\bar{e}_1 > 0$ and the banking equilibrium at $\theta^*(\bar{e}_1)$. If capital requirement is increased to $\bar{e}_2 > \bar{e}_1$, in the new equilibrium the fraction of risky banks decreases $\theta^*(\bar{e}_2) < \theta^*(\bar{e}_1)$.

**Proof.** See appendix.

Higher capital requirement forces risky bankers to contribute additional capital. Intuitively, the gains from risk-taking are reduced as the leverage becomes lower. If the fraction of risky banks is unchanged under the higher capital requirement, the expected payoff of risky banks is strictly lower than opportunity cost of capital. Note that safe banks are also forced to use a higher level of capital, whose cost may not be fully financed as well. Therefore the optimal contract of safe banks may become less attractive, making it less costly for risky banks to mimic. However, such savings from paying a cheaper contract are not sufficient to offset the reduction in payoff of risky banks.\footnote{Since safe banks always invest in relatively more efficient assets, the decrease in contract terms is relatively small. In addition, safe banks pay contracts in both state $G$ and state $B$, while risky banks only pay contracts in state $G$. Therefore the savings from paying a cheaper contract are even smaller and the details are shown in appendix.} Risky bankers would want to change to safe bankers or exit.

As a result, the fraction of risky banks must decrease in order to reach the new equilibrium. As the fraction of risky banks decreases, the size of potential bankruptcies and asset sales is reduced. On the other hand, the fraction of safe banks increases and the available liquidity increases. The price of illiquid asset $q_B$ will become less depressed, making safe banks less profitable in the $B$ state.
Therefore the consumption contract will become less attractive. Cheaper and cheaper contracts will eventually allow risky banks to receive the same payoff as the opportunity cost of capital.

As shown in Proposition 6, the fraction of risky banks $\theta^*(\bar{e})$ decreases as $\bar{e}$ increases. Denote $\bar{e}_H$ as the capital requirement such that $\theta^*(\bar{e}_H) = 0$. Under $\bar{e}_H$, the fraction of risky banks is zero and inefficient risk-taking is completely eliminated in the banking system.

**Remark** There always exists an optimal level of capital requirement $\bar{e}^* \leq \bar{e}_H$ such that consumer welfare is maximized. It may be optimal to completely eliminate risk-taking (Case A of Figure 1). Or the optimal capital requirement $\bar{e}^*$ may be such that $\bar{e}^* < \bar{e}_H$ and $\theta^*(\bar{e}^*) < 0$: it is optimal not to completely eliminate risk-taking in the banking system (Case B of Figure 1).

The x-axis represents the level of capital requirement and the y-axis represents the welfare of all consumers. The red line is the welfare of consumers under planner’s constrained efficient allocation $C^{**}$. The blue line is the welfare of consumers in the banking equilibrium with financial opacity. The left panel plots the situation in which welfare is maximized when risk-taking is completely eliminated. The right panel plots the situation in which welfare is maximized when risk-taking is not completely eliminated.

**Figure 1: Impact of Capital Requirements on Welfare**

Increasing capital requirement has two opposing effects. On one hand, a higher capital requirement can potentially improve welfare by reducing the number of risky banks in the economy. By increasing capital requirement to $\bar{e}_2 > \bar{e}_1$, there is an increase of $(\theta^*(\bar{e}_1) - \theta^*(\bar{e}_2))$ fraction of consumers that are with safe banks. By avoiding potential low consumption in the $B$ state, the welfare of these consumers can increase. On the other hand, a higher capital requirement could reduce welfare by forcing safe banks to utilize an undesirable level of capital. Additional resources have to be taken away from consumers to pay the cost of capital and consumption contract will be reduced for $(1 - \theta^*(\bar{e}_1))$ fraction of consumers.

The overall impact on welfare depends on the relative magnitude of each force and is illustrated using numerical simulations in the next section. Capital requirement should never exceed $\bar{e}_H$ as risky-taking is completely eliminated at $\bar{e}_H$. For any $\bar{e} > \bar{e}_H$, the banking equilibrium becomes the
banking equilibrium with only one type of bank, as in the case without financial opacity. Welfare strictly decreases as $\bar{e}$ increases as shown in proposition 5. Therefore there always exists an optimal level of capital requirement $\bar{e}^* \in (0, \bar{e}_H]$.  

4.2.1 Numerical Simulations

Two numerical examples are given to illustrate the impact of capital requirements on risk-taking and welfare. Table 1 contains values of parameters used in Example 4A and Example 4B. The exact functional forms of $u(c)$, $F(z)$, and $K(e)$ are presented in appendix.

<table>
<thead>
<tr>
<th>Table 1: Parameters</th>
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<tbody>
<tr>
<td>Aggregate state $i$</td>
</tr>
<tr>
<td>Liquidity Shock $j$</td>
</tr>
<tr>
<td>Cost of Capital</td>
</tr>
<tr>
<td>Capital Requirement</td>
</tr>
</tbody>
</table>

Example 4A describes a normal economic environment. The marginal net return of long asset investment decreases linearly from 20% to 0%. The risky asset always generates 20% in the good times. The marginal net cost of capital increases linearly from 10% to 20%. Example 4B describes an extremely poor economic environment. Long asset investment is not generating any positive return and all other parameters remain the same as in Example 4A. This is an effective way to illustrate the impact of undesirable capital requirements.

The simulation results are shown in Figure 2. In Example 4A, capital requirement is effective in reducing the number of risky banks. Recall that in equilibrium, the marginal return of long asset investment represents the gross return in the banking sector. Because long asset is reasonably productive, with a higher level of capital, safe bank’s contract terms do not reduce much. In order to achieve the new equilibrium, the fraction of risky banks reduces substantially for each unit increase in capital requirement. The autarky option is not included here and is actually not material. In reality, consumers face losses from private storage such as theft and flood. As a result, participation in the banking sector can always be made attractive to consumers by assuming the existence of some undesirable events when using private storage.

In Example 4B, the long asset is essentially unproductive. For each unit increase in capital, safe bank’s contract reduces substantially since safe banks do not have productive opportunity to invest. The reduction of the fraction of risky banks is very slow. Only a small fraction of consumers become better off from being with safe banks, but many existing safe bank consumers
suffer from lower level of consumption. Welfare attains its maximum way before risk-taking is completely eliminated. This example sheds an interesting light on the use of capital requirement during recession or stagnation. If equity capital is scarce and costly, and the economic outlook is dim as the return of regular investment is low, increasing capital requirements could result in welfare losses to consumers.

5 Conclusion

Motivated by the empirical features of the U.S. financial market in the 2007-2009 financial crisis, this paper studies the optimal contract and portfolio decisions of banks in an opaque banking system that can account for the heterogeneity in bank asset portfolio observed in the crisis. In the existence of financial opacity, inefficient risk-taking by a small number of banks is likely to exist in the banking system. The potential bankruptcies of risk-taking banks lead to a reduction in consumer welfare, but the resulting asset sales enrich healthier banks, which can then offer a more attractive contract that reduces the incentive for risk-taking. In equilibrium, the banking system is endogenously divided into risk-taking banks and safe, healthy banks and there exists an optimal level of capital requirement.

During the 2007-2009 financial crisis, the U.S. government undertook massive liquidity injections. Prior to the crisis, credit default swaps were widely used by healthy institutions to hedge risk and by risk-taking institutions to take risk. The possibility of introducing liquidity injections and CDS into this banking model provides promising options for future studies.
References


Appendix A

A3.1 Bank’s Optimization Problem without Financial Opacity

Taking prices \((q_G, q_B)\) as given, banks choose \((C, e, A)\) to maximize expected utility of typical consumer. The Lagrangian function is

\[
\mathcal{L} = \frac{1}{2} [\lambda_H u(d_H) + (1 - \lambda_H) u(c_H)] + \frac{1}{2} [\lambda_L u(d_L) + (1 - \lambda_L) u(c_L)]
\]

\[
+ \mu [\frac{1}{2} \pi F(1 + e - y) + \frac{y - \lambda_H d_H}{q_G} - (1 - \lambda_H) c_H] + \frac{1}{2} \pi [F(1 + e - y) + \frac{y - \lambda_L d_L}{q_G} - (1 - \lambda_L) c_L]
\]

\[
+ \frac{1}{2} (1 - \pi) [F(1 + e - y) + \frac{y - \lambda_H d_H}{q_B} - (1 - \lambda_H) c_H] + \frac{1}{2} (1 - \pi) [F(1 + e - y) + \frac{y - \lambda_L d_L}{q_B} - (1 - \lambda_L) c_L]
\]

\[- K(e^S)]

\[
+ \mu_{GH} [F(1 + e - y) + \frac{y - \lambda_G d_H}{q_G} - (1 - \lambda_H) c_H] + \mu_{GL} [F(1 + e - y) + \frac{y - \lambda_L d_L}{q_G} - (1 - \lambda_L) c_L]
\]

\[
+ \mu_{BH} [F(1 + e - y) + \frac{y - \lambda_H d_H}{q_B} - (1 - \lambda_H) c_H] + \mu_{BL} [F(1 + e - y) + \frac{y - \lambda_L d_L}{q_B} - (1 - \lambda_L) c_L]
\]

\[+ \mu_H (c_H - d_H) + \mu_L (c_L - d_L) + \omega (e - \bar{e})
\]

Take the first order conditions with respect to \((d_H, d_L)\) and \((c_H, c_L)\)

\[
u'(d_H) = \mu [\frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] + 2\mu_{GH} \frac{1}{q_G} + 2\mu_{BH} \frac{1}{q_B} + \frac{1}{2} \lambda_H \mu_H
\]

\[
u'(d_L) = \mu [\frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] + 2\mu_{GL} \frac{1}{q_G} + 2\mu_{BL} \frac{1}{q_B} + \frac{1}{2} \lambda_L \mu_L
\]

\[
u'(c_H) = \mu + 2\mu_{GH} + 2\mu_{BH} - \frac{1}{2} (1 - \lambda_H) \mu_H
\]

\[
u'(c_L) = \mu + 2\mu_{GL} + 2\mu_{BL} - \frac{1}{2} (1 - \lambda_L) \mu_L
\]

Take the first order conditions with respect to \(y\) and \(e\)

\[
\mu F'(1 + e - y) + \mu_{GH} + \mu_{GL} + \mu_{BH} + \mu_{BL} = \mu [\frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] + \frac{\mu_{GH} + \mu_{GL}}{q_G} + \frac{\mu_{BH} + \mu_{BL}}{q_B}
\]

\[
\mu F'(1 + e - y) + \mu_{GH} + \mu_{GL} + \mu_{BH} + \mu_{BL} + \omega = \mu K'(e)
\]

If none of the state resources constraints binds, the optimality conditions are shown in the main text. Assume capital requirement \(e > \bar{e}\) does not bind. The situation in which the capital requirement binds is studied in Appendix A.3.5. If some of the resources constraints could bind, the problem contains regular Kuhn-Tucker conditions and the optimal solutions are still well characterized. Suppose \(V_{GH} \geq 0\) binds, then \(\mu_{GH} > 0\). The optimality conditions become

\[
u'(d_H) = \mu [\frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] + 2\mu_{GH} \frac{1}{q_G} \quad \text{(A3.1.1)}
\]

\[
u'(d_L) = \mu [\frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] \quad \text{(A3.1.2)}
\]

\[
u'(c_H) = \mu + 2\mu_{GH} \quad \text{(A3.1.3)}
\]

\[
u'(c_L) = \mu \quad \text{(A3.1.4)}
\]

\[
\mu F'(1 + e - y) + \mu_{GH} = \mu [\frac{1}{q_G} + (1 - \pi) \frac{1}{q_B}] + \frac{\mu_{GH}}{q_G} \quad \text{(A3.1.5)}
\]

\[
\mu F'(1 + e - y) + \mu_{GH} = \mu K'(e) \quad \text{(A3.1.6)}
\]}
\[ V_{GH} = F(1 + e - y) + \frac{y - \lambda_G d_H}{q_G} - (1 - \lambda_H)c_H = 0 \]  
(\text{A3.1.7})

The optimal decisions \((d_H, d_L, c_H, c_L; e, y, q_B)\) and multipliers \((\mu, \mu_{GH})\) are characterized using equations (A3.1.1) to (A3.1.7), and (3.1) and (3.10). It is a system of 9 equations in 9 variables. Using the equations above

\[
u'(d_H) > u'(d_L) \quad \text{and} \quad u'(c_H) > u'(c_L)\]

\[ F'(1 + e - y) < K'(e)\]

In this case, \(d_H < d_L\) and \(c_H < c_L\). Consumptions are lower when facing a high liquidity shock \(H\). Also in order to smooth consumption across all states, banks use slightly more capital at a cost that cannot be fully financed. Using additional capital transfers resources to the resources-constrained state \(GH\) so that expected utility of consumer can be increased.

**A3.2 Proof of Proposition 1**

In the social planner’s problem, the optimal decisions \((d^*, c^*, e^*, y^*)\) are characterized by equations (2.4), (2.5), (2.9), and (2.10). In the banking equilibrium, when the resources constraints (3.2) and capital constraint (3.4) do not bind, the optimal contract has \(d^* = d^*_j\) and \(c^* = c^*_j\), for \(j = H, L\). The optimality conditions (3.8) and (3.9) in the bank’s problem are the same as the ones in the planner’s problem. Bank’s participation constraint (3.1) and liquidity condition (3.10) are equivalent to the resources constraints in the planner’s problem. The only additional equation that determines price \(q_B\) is equation (3.11), since banking equilibrium consists of prices \((q_G, q_B)\). Therefore the banking equilibrium allocation is the same as the constrained efficient allocation.

**A3.3 Proof of Lemma 1**

Consider any portfolio of risky banks \(A^R = (y^R, x^R, z^R)\) such that \(y^R > 0, x^R > 0,\) and \(z^R > 0\). Since bankers of risky banks only receive payoffs in the \(G\) state, the expected payoff of risky banks is

\[ E[V^R_{ij}((C^S, e^R, A^R)] = \pi[\frac{1}{2} V^R_{GH} + \frac{1}{2} V^R_{GL}]\]

\[ E[V^R_{ij}((C^S, e^R, A^R)] = \pi[F(z^R) + x^R R_G + y^R \frac{y - \lambda_G d_H}{q_G} - \frac{1}{2}(\lambda_H d^S_H + (1 - \lambda_H)c^S_H) - \frac{1}{2}(\lambda_L d^S_L + (1 - \lambda_L)c^S_L)]\]

Substituting \(q_G = 1\), the expected payoff becomes

\[ E[V^R_{ij}((C^S, e^R, A^R)] = \pi[F(z^R) + x^R R_G + y^R - \frac{1}{2}(\lambda_H d^S_H + (1 - \lambda_H)c^S_H) - \frac{1}{2}(\lambda_L d^S_L + (1 - \lambda_L)c^S_L)]\]

Suppose risky banks reduces the short asset investment by \(\epsilon > 0\) units and invest in the risky asset. The change in expected payoff is

\[ E[V^R_{ij}((C^S, e^R, (y^R - \epsilon, x^R + \epsilon, z^R))] - E[V^R_{ij}((C^S, e^R, (y^R, x^R, z^R))]\]

\[ = \pi[F(z^R) + (x^R + \epsilon) R_G + y^R - \epsilon] - \pi[F(z^R) + x^R R_G + y^R]\]

\[ = \pi \epsilon(R_G - 1) > 0 \quad \text{as} \quad R_G > 1\]

The change in expected payoff is always positive. Therefore risky banks can obtain a higher payoff.
by replacing investment in short asset by risky asset.

Similarly, suppose risky banks reduce the long asset investment by \( \epsilon > 0 \) units and invest in the risky asset. The change in expected payoff is

\[
E[V^R_{ij}((C^S, e^R, (y^R, x^R + \epsilon, z^R - \epsilon))] - E[V^R_{ij}((C^S, e^R, (y^R, x^R, z^R))] = \pi[F(z^R - \epsilon) + (x^R + \epsilon)R_G + y^R] - \pi[F(z^R) + x^R R_G + y^R] = \pi[F(z^R - \epsilon) - F(z^R) + \epsilon R_G] \geq \pi[-\epsilon F'(z^R - \epsilon) + \epsilon R_G] \quad \text{as} \quad F''(z) < 0, \ \forall z
\]

\[
= \pi\epsilon[R_G - F'(z^R - \epsilon)] > 0 \quad \text{as} \quad R_G > F'(z), \ \forall z
\]

Risky banks can obtain a higher payoff by replacing investment in long asset by risky asset. Therefore risky banks will invest all resources in the risky asset.

**A3.4 Proof of Lemma 2**

The proof follows by evaluating the total expected payoff of risky bankers at different capital levels. From Lemma 1, given \( e^R \), risky banks choose \( x^R = 1 + e^R \). The total expected payoff of risky bankers is

\[
E[V^R_{ij}((C^S, e^R, A^R))] + F_O(w - e^R) = \pi\left(\frac{1}{2}V_{GH}^R + \frac{1}{2}V_{GL}^R\right) + F_O(w - e^R)
\]

\[
= \pi[(1 + e^R)R_G - \frac{1}{2}(\lambda_H d_H^S + (1 - \lambda_H) c_H) - \frac{1}{2}(\lambda_L d_L^S + (1 - \lambda_L) c_L)] + F_O(w - e^R)
\]

Suppose risky bankers reduce capital level by \( \epsilon > 0 \). The change in total expected payoff is

\[
E[V^R_{ij}((C^S, e^R - \epsilon, (0, 1 + e^R - \epsilon, 0)))] - E[V^R_{ij}((C^S, e^R, (0, 1 + e^R, 0)))] = \pi[(1 + e^R - \epsilon)R_G - (1 + e^R)R_G] + F_O(w - e^R + \epsilon) - F_O(w - e^R) = F_O(w - e^R + \epsilon) - F_O(w - e^R) - \pi R_G \epsilon
\]

Since \( F''_O(w - e) < 0 \), \( F_O(w - e^R + \epsilon) - F_O(w - e^R) > F''_O(w - e^R + \epsilon) \epsilon \). Therefore the change

\[
F_O(w - e^R + \epsilon) - F_O(w - e^R) - \pi R_G \epsilon > F''_O(w - e^R + \epsilon) \epsilon - \pi R_G \epsilon = \epsilon[F''_O(w - e^R + \epsilon) - \pi R_G] > 0 \quad \text{as} \quad F''_O > 1 \quad \text{and} \quad \pi R_G < 1.
\]

Therefore risky banks will choose the lowest possible level of capital.

**A3.5 Safe Bank’s Optimization Problem with Financial Opacity**

Given prices \( (q_G, q_B) \), safe banks choose \( (C^S, e^S, A^S) \) to maximize expected utility of typical consumer. The Lagrangian function is

\[
\mathcal{L} = \frac{1}{2}\pi[\lambda_H u(d_H^S) + (1 - \lambda_H) u(c_H^S)] + \frac{1}{2}\pi[\lambda_L u(d_L^S) + (1 - \lambda_L) u(c_L^S)] +(1 - \theta)\frac{1}{2}(1 - \pi)[\lambda_H u(d_H^S) + (1 - \lambda_H) u(c_H^S)] + (1 - \theta)\frac{1}{2}(1 - \pi)[\lambda_L u(d_L^S) + (1 - \lambda_L) u(c_L^S)] + \theta(1 - \pi)u(d^R)
\]

\[
+ \mu[\frac{1}{2}\pi[F(1 + e^S - y^S) + y^S - \lambda_H d_H^S] - (1 - \lambda_H) e_H^S] + \frac{1}{2}\pi[F(1 + e^S - y^S) + y^S - \lambda_L d_L^S] - (1 - \lambda_L) c_L^S]
\]
\[ + \frac{1}{2}(1-\pi)[F(1+e^S - y^S) + \frac{y^S - \lambda_H d^S_H}{q_B} - (1-\lambda_H)c^S_H] + \frac{1}{2}(1-\pi)[F(1+e^S - y^S) + \frac{y^S - \lambda_L d^S_L}{q_B} - (1-\lambda_L)c^S_L] - K(e^S) \]

\[ + \mu_{GH}[F(1+e^S - y^S) + \frac{y^S - \lambda_H d^S_H}{q_B} - (1-\lambda_H)c^S_H] + \mu_{GL}[F(1+e^S - y^S) + \frac{y^S - \lambda_L d^S_L}{q_B} - (1-\lambda_L)c^S_L] \]

\[ + \mu_{BH}[F(1+e^S - y^S) + \frac{y^S - \lambda_H d^S_H}{q_B} - (1-\lambda_H)c^S_H] + \mu_{BL}[F(1+e^S - y^S) + \frac{y^S - \lambda_L d^S_L}{q_B} - (1-\lambda_L)c^S_L] \]

Take the first order conditions with respect to \((d^S_H, d^S_L)\) and \((c^S_H, c^S_L)\)

\[ u'(d^S_H)[\frac{1}{2} \pi + (1-\theta)\frac{1}{2}(1-\pi)] = \mu[\frac{1}{2} \pi \frac{1}{q_G} + \frac{1}{2}(1-\pi)\frac{1}{q_B}] + 2\mu_{GH} \frac{1}{q_G} + 2\mu_{BH} \frac{1}{q_B} + \frac{1}{2}\lambda_H \mu_H \]

\[ u'(d^S_L)[\frac{1}{2} \pi + (1-\theta)\frac{1}{2}(1-\pi)] = \mu[\frac{1}{2} \pi \frac{1}{q_G} + \frac{1}{2}(1-\pi)\frac{1}{q_B}] + 2\mu_{GL} \frac{1}{q_G} + 2\mu_{BL} \frac{1}{q_B} + \frac{1}{2}\lambda_L \mu_L \]

\[ u'(c^S_H)[\frac{1}{2} \pi + (1-\theta)\frac{1}{2}(1-\pi)] = \frac{1}{2} \mu + 2\mu_{GH} + 2\mu_{BH} - \frac{1}{2}(1-\lambda_H) \mu_H \]

\[ u'(c^S_L)[\frac{1}{2} \pi + (1-\theta)\frac{1}{2}(1-\pi)] = \frac{1}{2} \mu + 2\mu_{GL} + 2\mu_{BL} - \frac{1}{2}(1-\lambda_L) \mu_L \]

Take the first order conditions with respect to \(y^S\) and \(e^S\)

\[ \mu F'(1+e^S - y^S) + \mu_{GH} + \mu_{GL} + \mu_{BH} + \mu_{BL} = \mu[\frac{1}{2} \pi \frac{1}{q_G} + (1-\pi)\frac{1}{q_B}] + \frac{\mu_{GH} + \mu_{GL}}{q_G} + \frac{\mu_{BH} + \mu_{BL}}{q_B} \]

\[ \mu F'(1+e^S - y^S) + \mu_{GH} + \mu_{GL} + \mu_{BH} + \mu_{BL} + \omega = \mu K'(e^S) \] (A3.5.1)

Since \(\omega > 0\), the marginal return of long asset investment is lower than the marginal cost of capital. Since \(F'(z) < 0\) and \(K''(e) > 0\), the optimal capital is equal to the capital requirement \(e^S = \tilde{e}\). Together with equations (3.21), (3.28), (A3.5.1), (3.30), and (3.31), the optimal solutions \((d^*, c^*, e^*, y^*; q_B)\) and multiplier \(\omega\) are fully characterized.

**A3.6 Proof of Lemma 3**

The proof is to show that there exists a \(\theta_2 \in (0,1)\) such that \(E[V^{R}_{ij}((C^S(\theta_2), e^R, A^R)] < K(e^R)\). Let \(\theta = 1 - \epsilon\), where \(\epsilon > 0\) is very small and is the measure of safe banks in the economy. By Lemma 1 and Lemma 2, risky banks choose \(x^R = 1 + \tilde{e}\) in the risky asset. The aggregate level of risky asset is \(\theta(1 + \tilde{e})\). Substitute \(\theta = (1 - \epsilon)\) into the market clearing condition (3.31) at \(B\)

\[ \epsilon(y^S - \frac{1}{2} \lambda_H d^S_H - \frac{1}{2} \lambda_L d^S_L) = (1-\epsilon)(1+\tilde{e})R_B q_B \] (A3.6.1)

therefore \(q_B\) can be expressed as

\[ q_B = \frac{\epsilon(y^S - \frac{1}{2} \lambda_H d^S_H - \frac{1}{2} \lambda_L d^S_L)}{(1-\epsilon)(1+\tilde{e})R_B} \] (A3.6.2)

As \(\epsilon \to 0\), \((1-\epsilon) \to 1\). The numerator \((y^S - \frac{1}{2} \lambda_H d^S_H - \frac{1}{2} \lambda_L d^S_L)\) is bounded, since \(y^S \leq (1 + e^S)\) and \(d^S_j \geq 0\). The denominator \((1-\epsilon)(1+\tilde{e})R_B \to (1+\tilde{e})R_B\). Therefore
\[
q_B = \epsilon (y^S - \frac{1}{2} \lambda_H d_H^S - \frac{1}{2} \lambda_L d_L^S) \frac{1}{(1-\epsilon)(1+\epsilon)R_B} \to 0 \quad \text{as} \quad \epsilon \to 0
\]

Note that it is not possible to have \((y^S - \frac{1}{2} \lambda_H d_H^S - \frac{1}{2} \lambda_L d_L^S)\) increasing to \(\infty\) to cancel the decreasing effect of \(\epsilon\). Assume by contradiction that \((y^S - \frac{1}{2} \lambda_H d_H^S - \frac{1}{2} \lambda_L d_L^S) \to \infty\) as \(\epsilon \to 0\). Since consumption terms \(d_j^S\) are bounded below by 0, it must be the case that \(y^S \to \infty\) and safe banks need to have \(e^S \to \infty\) to fund the infinite amount of investment in liquidity. However, this is not possible since \(e^S \in [\bar{c}, w]\). Also this violates safe bank’s optimality condition for capital level \(e^S\)

\[
K'(e^S) = \pi \frac{1}{q_G} + (1 - \pi) \frac{1}{q_B} = \frac{\pi}{q} + \frac{1 - \pi}{q_B}
\]

If \(q_B\) is bounded below, then the right-hand-side return of short asset investment is bounded above. Since \(K''(e) > 0\), the left-hand-side marginal cost of capital is increasing. As \(e^S \to \infty\), \(K'(e^S)\) becomes sufficiently large and this optimality condition will fail. Therefore \(q_B \to 0\) as \(\theta \to 1\).

As \(q_B\) is very small when \(\theta\) is large, this deeply discounted asset price gives safe banks lots of resources at \(t = 2\) in the \(B\) state. Safe banks will be able to offer a very attractive contract \(C^S\). Mimicking this expensive contract in the \(G\) state will give risky banks a very low expected payoff \(E[V_{ij}^G]\). First to notice that, when \(q_B\) is small enough, \(\frac{1}{q_B}\) is large enough such that

\[
\frac{\pi}{q_G} + \frac{1 - \pi}{q_B} > F'(z^S) \quad \text{for} \quad z^S \approx 0
\]

This is because by assumption \(F'(z) < R_G, \forall z > 0\). This means the marginal return on short asset investment is higher than on long asset investment for all units. Therefore safe banks will choose the entire portfolio in the short assets \(y^S = 1 + e^S\).

Competition among safe banks will force them to increase the contract terms \((d_j^S, c_j^S)\). Since \(y^S = 1 + e^S\), the expected payoff of safe banks becomes

\[
E[V_{ij}^S] = (1 + e^S)\left(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}\right) - \left(\frac{1}{2} \lambda_H d_H^S - \frac{1}{2} \lambda_L d_L^S\right)\left(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}\right) - \frac{1}{2} (1 - \lambda_H) c_H^S - \frac{1}{2} (1 - \lambda_L) c_L^H
\]

In equilibrium, the expected payoff of safe banks should be equal to opportunity cost of capital

\[
(1 + e^S)\left(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}\right) - \left(\frac{1}{2} \lambda_H d_H^S - \frac{1}{2} \lambda_L d_L^S\right)\left(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}\right) - \frac{1}{2} (1 - \lambda_H) c_H^S - \frac{1}{2} (1 - \lambda_L) c_L^H = K(e^S)
\]

Isolating the \(c_j^S\) terms

\[
\frac{1}{2} (1 - \lambda_H) c_H^S + \frac{1}{2} (1 - \lambda_L) c_L^H = (1 + e^S - \frac{1}{2} \lambda_H d_H^S - \frac{1}{2} \lambda_L d_L^S)\left(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}\right) - K(e^S) \quad (A3.6.3)
\]

The last step is to show that as \(q_B\) becomes very small, either \(c_j^S\) or \(d_j^S\) is not bounded above. Suppose both \(d_j^S\) and \(c_j^S\) are bounded above. As \(q_B \to 0\), \((\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}) \to \infty\). The right-hand-side must be increasing since \(e^S\) is bounded in the interval \([\bar{c}, w]\). So \(c_j^S\) are increasing and cannot be bounded above and therefore it is a contradiction. The contract is becoming prohibitively expensive for risky banks to mimic as \(c_j^S\) is unbounded. In fact, because of the first order condition of consumption smoothing between \(d_j^S\) and \(c_j^S\), consumption in both periods will likely increase. The contract terms \(d_j^S\) and \(c_j^S\) will be very high as \(q_B \to 0\). Therefore the expected payoff of risky banks will be smaller than the opportunity cost of capital.

A3.7 Proof of Lemma 4
Fix any $\theta \in [0,1)$ and given $(q_G, q_B)$, safe banks choose $(C^S, e^S, A^S)$ to maximize the expected utility of consumer. The constraints of the optimization problem are

$$E[V_{ij}^S|(C^S, e^S, A^S)] = K(e^S)$$

$$V_{ij}^S \geq 0$$

$$d_j^S \leq c_j^S$$

$$e^S \geq \bar{e}$$

It is necessary to show that the constraint correspondence is continuous. Denote set $Q = \{(q_G, q_B) \in R_+ \times R_+\}$ as the set of all possible pairs of prices $(q_G, q_B)$. Denote set $O = \{(C^S, e^S, A^S) \in R_+^4 \times R_+\}$ as the set of all possible choices of safe banks. The constraint correspondence $\varphi$ is defined as $\varphi : Q \Rightarrow O$. Since $x^S = 0$ for safe banks, $A^S$ can be replaced by $y^S$. I show that the constraint correspondence $\varphi$ is both upper hemicontinuous (uhc) and lower hemicontinuous (lhc).

For convenience, the superscript $S$ is omitted in the following expressions.

The first part is to prove that the correspondence $\varphi$ is lhc using the sequential characterization. The correspondence $\varphi : Q \Rightarrow O$ is lower hemicontinuous if and only if for any sequence $(q_{Gn}, q_{Bn}) \in Q$ for $n \in N$ such that $\lim_{n \to \infty} (q_{Gn}, q_{Bn}) = (q_G, q_B) \in Q$ and $(C, e, y) \in \varphi((q_G, q_B))$, there exists a sequence $(C_n, e_n, y_n) \in \varphi((q_{Gn}, q_{Bn}))$ for $n \in N$ such that $\lim_{n \to \infty} (C_n, e_n, y_n) = (C, e, y)$.

Let $(q_G0, q_B0)$ and $(C_0, e_0, y_0)$ be such that $(C_0, e_0, y_0) \in \varphi((q_G0, q_B0))$, i.e. satisfying all constraints (A3.7.1) to (A3.7.4). The proof is to construct a sequence $(C_n, e_n, y_n)$ where $C_n = (d_{Hn}, d_{Ln}, c_{Hn}, c_{Ln})$ such that $(C_n, e_n, y_n) \in \varphi((q_{Gn}, q_{Bn}))$ and $\lim_{n \to \infty} (C_n, e_n, y_n) = (C_0, e_0, y_0)$.

Consider a sequence of $(q_{Gn}, q_{Bn})$ with $\lim_{n \to \infty} (q_{Gn}, q_{Bn}) = (q_G0, q_B0)$. Depending on the situations, the sequence is constructed appropriately to satisfy all the constraints.

**Case 1.** At the limit points $(q_G0, q_B0)$ and $(C_0, e_0, y_0)$, $E[V_{ij}^S|(C_0, e_0, y_0; q_G0, q_B0)] = K(e_0)$ and

$$V_{ij}(C_0, e_0, y_0; q_G0, q_B0) > 0, \quad d_j0 < c_j0, \quad e_0 > \bar{e}$$

The participation constraint is held with equality, and all other constraints are held with strictly inequality. Since $\lim_{n \to \infty} (q_{Gn}, q_{Bn}) = (q_G0, q_B0)$, there exits a sufficiently large $N$ such that for $n \geq N$

$$V_{ij}(C_0, e_0, y_0; q_{Gn}, q_{Bn}) > 0, \quad d_jn < c_jn, \quad e_n > \bar{e}$$

For the equality constraint $E[V_{ij}^S|(C_0, e_0, y_0; q_G0, q_B0)] = K(e_0)$, define $\delta$ as

$$\delta = E[V_{ij}^S|(C_0, e_0, y_0; q_G0, q_B0)] - K(e_0)$$

(i) Along the sequence $(q_{Gn}, q_{Bn})$, if $\delta < 0$: the expected payoff is less than cost of capital, consumption contract should be reduced. Therefore define the following sequence for $n > N$

$$(C_n, e_n, y_n) = (C_n, e_0, y_0) \quad \text{with} \quad C_n = (d_{Hn}, d_{Ln}, c_{Hn}, c_{Ln})$$

such that

$$d_jn = \min[d_j0, d_j0 + \frac{\delta}{2(\lambda_H + \lambda_L)}(\frac{e_j0}{q_{Gn}} + \frac{\delta}{q_{Bn}} + (1 - \frac{1}{2}(\lambda_H + \lambda_L)))$$

for $j = H, L$

$$c_jn = \min[c_j0, c_j0 + \frac{\delta}{2(\lambda_H + \lambda_L)}(\frac{e_j0}{q_{Gn}} + \frac{\delta}{q_{Bn}} + (1 - \frac{1}{2}(\lambda_H + \lambda_L)))$$

for $j = H, L$.
Condition \( n > N \) ensures that the constraints \((A3.7.2),(A3.7.2),(A3.7.4)\) are always satisfied with strict inequality. Both early and late period consumption are adjusted downwards simultaneously along the sequence so that the participation constraint \((A3.7.1)\) is satisfied with equality. Note that after the downward adjustments of \((d_{j0},c_{j0})\), the resources constraints \(V_{ij} > 0\) always hold.

(ii) Along the sequence \((q_{Gn}, q_{Bn})\), if \(\delta > 0\): the expected payoff is greater than cost of capital, capital level should be increased. Therefore define the following sequence for \(n > N\)
\[
(C_n, e_n, y_n) = (C_0, e_n, y_0) \quad \text{with} \quad e_n = \max[e_0, \hat{e}] \quad \text{such that}
\]
\[
F(1+\hat{\epsilon} - y_0) + \left(\frac{\pi}{q_{Gn}} + \frac{1}{q_{Bn}}\right)(y_0 - \frac{1}{2}(\lambda_H d_{H0} + \lambda_L d_{L0})) - \frac{1}{2}(1 - \lambda_H) c_{H0} - \frac{1}{2}(1 - \lambda_L) c_{L0} = K(\hat{\epsilon})
\]
Since \(\frac{\partial F(1+\epsilon - y)}{\partial \epsilon} < 0\) and \(\frac{\partial K(\epsilon)}{\partial \epsilon} > 0\), an increase in \(\epsilon\) reduces \(\delta\). There exists an \(\hat{\epsilon}\) such that the constraint \(E[V_{ij}](C_0, \hat{\epsilon}, y_0; q_{Gn}, q_{Bn}) = K(\hat{\epsilon})\) is satisfied with equality. Note that upward adjustment of \(e_0\) ensures that the constraints \(V_{ij} > 0\) are satisfied. In this case, we cannot adjust consumption \((d_{j0},c_{j0})\) upwards since this could make state resources constraints bind.

Case 2. At the limit points \((q_{G0}, q_{B0})\) and \((C_0, e_0, y_0)\), \(E[V_{ij}](C_0, e_0, y_0; q_{G0}, q_{B0}) = K(e_0)\) and
\[
V_{ij}(C_0, e_0, y_0; q_{Gn}, q_{Bn}) = 0 \quad \text{for one} \quad ij, \quad d_j < c_j, \quad e_n > \bar{e}
\]
One of the state resources constraints is held with equality. The strategy is to construct a sequence that always ensures the equality constraint \(E[V_{ij}] = K(e_0)\) and at the same time, makes \(V_{ij} \geq 0\). Assume without loss of generality that constraint \(V_{GH} = 0\). Since \(\lim_{n \to \infty}(q_{Gn}, q_{Bn}) = (q_{G0}, q_{B0})\), there exits a sufficiently large \(N\) such that for \(n \geq N\)
\[
V_{ij}(C_0, e_0, y_0; q_{Gn}, q_{Bn}) > 0 \quad \text{for} \quad ij \in \{GL, BH, BL\} \quad \text{and}
\]
\[
d_j < c_j, \quad e_n > \bar{e}
\]
Define \(\delta\) and \(\epsilon\) as follows
\[
\delta = E[V_{ij}](C_0, e_0, y_0; q_{Gn}, q_{Bn}) - K(e_0)
\]
\[
\epsilon = V_{GH}(C_0, e_0, y_0; q_{Gn}, q_{Bn})
\]

(i) Along the sequence \((q_{Gn}, q_{Bn})\), if \(\delta < 0\) and \(\epsilon > 0\): the expected payoff is less than cost of capital and state resources constraint in \(GH\) is satisfied. Consumption should be reduced. Therefore define the following sequence for \(n > N\)
\[
(C_n, e_n, y_n) = (C_0, e_0, y_0) \quad \text{with} \quad C_n = (d_{Hn}, d_{Ln}, c_{Hn}, c_{Ln}) \quad \text{such that}
\]
\[
d_j = \min[d_{j0}, d_{j0} + \frac{1}{2}(\lambda_H + \lambda_L)(\frac{\pi}{q_{Gn}} + \frac{1}{q_{Bn}})/(1 - \frac{1}{2}(\lambda_H + \lambda_L))]
\]
\[
c_j = \min[c_{j0}, c_{j0} + \frac{1}{2}(\lambda_H + \lambda_L)(\frac{\pi}{q_{Gn}} + \frac{1}{q_{Bn}})/(1 - \frac{1}{2}(\lambda_H + \lambda_L))]
\]
The consumption is adjusted downwards so that the constraint \(E[V_{ij}](C_n, e_0, y_0; q_{Gn}, q_{Bn}) = K(e_0)\) is satisfied with equality. Note that after the downward adjustment of consumption, the constraints \(V_{ij} > 0\) always hold for all \(ij\).

(ii) Along the sequence \((q_{Gn}, q_{Bn})\), if \(\delta < 0\) and \(\epsilon < 0\): the expected payoff is less than cost of
consumption until $V_{GH} \geq 0$. Define the following sequence for $n > N$

$$(C_n, e_n, y_n) = (C_n, e_n, y_0) \quad \text{with} \quad C_n = (d_{Hn}, d_{Ln}, c_{Hn}, c_{Ln})$$

Also define $\hat{C}_n = (\hat{d}_{Hn}, \hat{d}_{Ln}, \hat{c}_{Hn}, \hat{c}_{Ln})$ such that

$$\hat{d}_{jn} = \min[d_{jn}, d_{jn} + \frac{\epsilon}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$

$$\hat{c}_{jn} = \min[c_{jn}, c_{jn} + \frac{1}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$

Second, at this new consumption level, if $E[V_{ij}](\hat{C}_n, e_0, y_0; q_{Gn}, q_{Bn}) - K(e_0) > 0$, increase capital $c$.

$$c_{jn} = \hat{c}_{jn} \quad \text{and} \quad e_n = \max[e_0, \hat{e}]$$

such that

$$\hat{e} \in \{E[V][(y_0, \hat{e}, C_n; q_{Gn}, q_{Bn})] = K(\hat{e}_n)\}$$

If $E[V_{ij}](\hat{C}_n, e_0, y_0; q_{Gn}, q_{Bn}) - K(e_0) < 0$, further reduce consumption.

$$d_{jn} = \min[d_{jn}, d_{jn} + \frac{\epsilon}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$

$$c_{jn} = \min[c_{jn}, c_{jn} + \frac{1}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$

In the first step the consumption is adjusted downwards so that the constraint $V_{ij} \geq 0$ is satisfied. In the second step, at this new consumption level, capital is adjusted upward or consumption further downward to ensure the participation constraint is held with equality. In the second step, the state resources constraints $V_{ij} > 0$ always hold for all $ij$, including $GH$.

(iii) Along the sequence $(g_{Gn}, q_{Bn})$, if $\delta > 0$ and $\epsilon > 0$: the expected payoff is greater than cost of capital and state resources constraint is satisfied. This is the same as case 2(i) so increase capital $e$. Therefore define the following sequence for $n > N$

$$(C_n, e_n, y_n) = (C_0, e_n, y_0) \quad \text{with} \quad e_n = \max[e_0, \hat{e}]$$

such that

$$\hat{e} \in \{E[V_{ij}][(0, \hat{e}, y_0; q_{Gn}, q_{Bn})] = K(\hat{e}_n)\}$$

(iv) Along the sequence $(g_{Gn}, q_{Bn})$, if $\delta > 0$ and $\epsilon < 0$: the expected payoff is greater than cost of capital and state resources constraint is not satisfied. Two steps are needed. First, reduce consumption until $V_{GH} \geq 0$. At this point, $\delta$ takes an even higher value. Then increase capital $e$ to make the quality constraint hold. Define the following sequence for $n > N$

$$(C_n, e_n, y_n) = (C_n, e_n, y_0) \quad \text{with} \quad C_n = (d_{Hn}, d_{Ln}, c_{Hn}, c_{Ln})$$

Also define $\hat{C}_n = (\hat{d}_{Hn}, \hat{d}_{Ln}, \hat{c}_{Hn}, \hat{c}_{Ln})$ such that

$$\hat{d}_{jn} = \min[d_{jn}, d_{jn} + \frac{\epsilon}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$

$$\hat{c}_{jn} = \min[c_{jn}, c_{jn} + \frac{1}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$

At this new consumption level, since $E[V_{ij}](\hat{C}_n, e_0, y_0; q_{Gn}, q_{Bn}) - K(e_0) > 0$, increase capital $e$. Therefore define the following sequence for $n > N$

$$(C_n, e_n, y_n) = (C_0, e_n, y_0) \quad \text{with} \quad C_n = (d_{Hn}, d_{Ln}, c_{Hn}, c_{Ln})$$

Also define $\hat{C}_n = (\hat{d}_{Hn}, \hat{d}_{Ln}, \hat{c}_{Hn}, \hat{c}_{Ln})$ such that

$$\hat{d}_{jn} = \min[d_{jn}, d_{jn} + \frac{\epsilon}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$

$$\hat{c}_{jn} = \min[c_{jn}, c_{jn} + \frac{1}{2}(\lambda_H + \lambda_L)(\frac{C_b}{q_{Gn}} + \frac{\delta}{q_{Bn}}) + (1 - \frac{1}{2})(\lambda_H + \lambda_L)]$$

for $j = H, L$
It is sufficient to show that level $e$.

$$c_{jn} = \hat{c}_{jn} \quad \text{and} \quad e_n = \max[e_0, \hat{e}] \quad \text{such that}$$

$$\hat{e}_n \in \{ E[V][(y_0, \hat{e}_n, C_n, q_{Gn}, q_{Bn})] = K(\hat{e}_n) \}$$

Under this construction, there is a sequence $(C_n, e_n, y_n)$ where $C_n = (d_{Hn}, d_{Ln}, c_{Hn}, c_{Ln})$ such that for $n > N$, $(C_n, e_n, y_n) \in \varphi((q_{Gn}, q_{Bn}))$ and $\lim_{n \to \infty}(C_n, e_n, y_n) = (C_0, e_0, y_0)$. Therefore the constraint correspondence is lower hemicontinuous.

By assumption the correspondence $\varphi$ is compact-valued. Since the constraints are defined by equality or weak inequality, the graph of the constraint correspondence $\varphi$ on any subset of $Q$ is compact, then the correspondence is upper hemicontinuous. Therefore the maximum theorem can be applied and the optimal solution $(C^*, e^*, A^*)$ is nonempty, compact-valued, and upper hemicontinuous. Since the objective function is concave, the optimal solution is compact, then the correspondence is upper hemicontinuous. Since the objective function is concave, the optimal solution $C^*$, $e^*$, $A^*$ is a single-valued correspondence and therefore is a continuous function of $q_B$. Using the market clearing condition (3.31) in the $B$ state, $q_B$ is a continuous function of $\theta$ since $(C^*, e^*, A^*)$ are continuous functions of $q_B$. Therefore $(C^*, e^*, A^*)$ and $q_B$ are continuous functions of $\theta$. Therefore the expected payoff of risky banks $E[V_{ij}^R]$ is continuous in $\theta$.

**A3.8 Proof of Proposition 4**

By construction, both safe bankers and risky bankers have chosen the optimal portfolio decisions. This part of proof is to show that a bank cannot obtain a higher payoff by receiving payoffs in the $B$ state and defaulting in the $G$ state. At the equilibrium candidate under $\theta^*$, after offering contract $C_S^*$, a bank that invests all resources in the short asset will receive strictly lower payoff than cost of capital.

The expected payoff of a bank choosing all liquidity $A^m = (y^m = 1 + \hat{e}, 0, 0)$ is

$$E[V_{ij}^m] = (1 + \hat{e}) \frac{1 - \pi}{q_B} + (1 - \pi)[\frac{1}{2}(\lambda_H + \lambda_L)d + (1 - \frac{1}{2}(\lambda_H + \lambda_L))c]$$

(3.3.8.1)

It is sufficient to show that $E[V_{ij}^m] - K(\hat{e}) < 0$.

For the equilibrium candidate under $\theta^*$, the expected payoff of safe and risky banks are

$$E[V_{ij}^S] = F(z) + y^S(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}) - \frac{1}{2}(\lambda_H + \lambda_L)d + (1 - \frac{1}{2}(\lambda_H + \lambda_L))c = K(e)$$

(3.3.8.2)

$$E[V_{ij}^R] = \pi(1 + \hat{e}) - \pi[\frac{1}{2}(\lambda_H + \lambda_L)d + (1 - \frac{1}{2}(\lambda_H + \lambda_L)c) = K(\hat{e})$$

(3.3.8.3)

Take difference between (3.3.8.2) and (3.3.8.3) and solve for $(1 - \pi)[\frac{1}{2}(\lambda_H + \lambda_L)d + (1 - \frac{1}{2}(\lambda_H + \lambda_L)c)]$

$$= F(z) + y^S(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}) - \pi(1 + \hat{e}) - K(e) + K(\hat{e})$$

(3.3.8.4)

Substitute into $E[V_{ij}^m] - K(\hat{e})$

$$E[V_{ij}^m] - K(\hat{e}) = (1 + \hat{e}) \frac{1 - \pi}{q_B} - F(z) - y^S(\frac{\pi}{q_G} + \frac{1 - \pi}{q_B}) + \pi(1 + \hat{e}) + K(e) - K(\hat{e})$$

Since $F''(z) < 0$, $F'(z)F''(z)z = (\frac{\pi}{q_G} + \frac{1 - \pi}{q_B})z^2$ and use $q_G = 1.$
The first order condition cannot be satisfied. Therefore, must have: 

\[ E[V^m_{ij}] - K(\bar{e}) < (1 + \bar{e})(\pi + \frac{1 - \pi}{\eta B}) - (z^S + y^S)(\pi + \frac{1 - \pi}{\eta B}) + K(e^S) - K(\bar{e}) \]

\[ E[V^m_{ij}] - K(\bar{e}) < (\bar{e} - e^S)(\pi + \frac{1 - \pi}{\eta B}) + K(e^S) - K(\bar{e}) \quad \text{as} \quad z^S + y^S = 1 + e^S \quad (A3.8.5) \]

Case (i): if capital requirement is binding \( e^S = \bar{e} \), then \( E[V^m_{ij}] - K(\bar{e}) < 0 \). Case (ii): if capital requirement is not binding \( e^S > \bar{e} \), then \( F'(z^S) = K'(e^S) \). Since \( K''(e) > 0 \)

\[ K(e^S) - K(\bar{e}) < K'(e^S)(e^S - \bar{e}) \]

Substitute into (A3.8.5) and use \( K'(e^S) = (\pi + \frac{1 - \pi}{\eta B}) \)

\[ E[V^m_{ij}] - K(\bar{e}) < (\bar{e} - e^S)K'(e^S) + K'(e^S)(e^S - \bar{e}) = 0 \]

Therefore a bank receives a strictly lower expected payoff by investing all resources in liquidity.

**A4.1 Proof of Lemma 5**

Without financial opacity, consider any strictly binding capital requirement \( \bar{e} > e^*_0 \). In this case, the optimal capital level chosen by banks is \( e^*(\bar{e}) = \bar{e} \). In the following proof, the optimal solutions under \( \bar{e} \) are sometimes written as \( y^* \) for short instead of \( y^*(\bar{e}) \). Define \( \delta \) as the increase in capital

\[ \delta = e^*(\bar{e}) - e^*_0 = \bar{e} - e^*_0 \]

Substitute \( y^*(\bar{e}) = \frac{1}{2}(\lambda_H + \lambda_L)d^*(\bar{e}) \) into the expected payoff of banks

\[ E[V_{ij}](C^*(\bar{e}), \bar{e}, A^*(\bar{e})) = F(1 + \bar{e} - y^*(\bar{e})) - (1 - \frac{1}{2}(\lambda_H + \lambda_L))c^*(\bar{e}) = K(\bar{e}) \]

Case (i): the proof is to show that as the amount of investable resources increases by \( \delta > 0 \), long asset investment must increase \( z^*(\bar{e}) > z^*_0 \). Suppose for contradiction that \( z^* = 1 + \bar{e} - y^* \leq z^*_0 \): investment in long asset is reduced. This gives \( F(z^*) \leq F(z^*_0) \), as \( F'(z) > 0 \). Also \( K(\bar{e}) > K(e^*_0) \).

The consumption \( c^* \) should be lower since

\[ c^* = \frac{F(z^*) - K(\bar{e})}{1 - \frac{1}{2}(\lambda_H + \lambda_L)} < \frac{F(z^*_0) - K(e^*_0)}{1 - \frac{1}{2}(\lambda_H + \lambda_L)} = c^*_0 \quad (A4.1.1) \]

At the same time, by assumption as \( z^* \leq z^*_0 \), \( y^* = 1 + \bar{e} - z^* \geq y^*_0 + \delta \). The consumption \( d^* \) should be higher since

\[ d^* = \frac{\frac{y^*}{2(\lambda_H + \lambda_L)}}{\frac{y^*_0}{2(\lambda_H + \lambda_L)}} = d^*_0 \]

However, since \( z^* \leq z^*_0 \), \( F'(z^*) \geq F'(z^*_0) \) as \( F''(z) < 0 \). This is in contradiction to the first order condition relating \( d^* \) and \( c^* \)

\[ u'(d^*) < F'(z^*)u'(c^*) \]

The first order condition cannot be satisfied. Therefore must have \( z^*(\bar{e}) > z^*_0 \): investment in long asset must strictly increase and therefore \( F'(z^*(\bar{e})) < F'(z^*_0) \).

Case (ii): the proof is to show that as the amount of investable resources increases by \( \delta > 0 \), consumption for late consumers decreases \( c^*(\bar{e}) < c^*_0 \). Suppose for contradiction that \( c^*(\bar{e}) \geq c^*_0 \). According to (A4.1.1), it must be the case that

\[ F(z^*) - K(\bar{e}) > F(z^*_0) - K(e^*_0) \]

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Since $F''(z) < 0$ and $K''(e) > 0$, a necessary condition is that $z^* > z^* + \delta$. This means short asset investment decreases $y^* < y^*_0$. As a result, $d^* < d^*_0$. This again violates the optimality condition for consumption

$$u'(d^*) = F'(z^*_0)u'(c^*_0) > F'(z^*)u'(c^*)$$

Therefore late period consumption must decrease $c^*(\bar{e}) < c^*_0$. Similarly, for any $\bar{e}_1$ and $\bar{e}_2$ such that $\bar{e}_2 > \bar{e}_1 \geq e^*_0$, replace $e^*_0$ by $\bar{e}_1$ and replace $\bar{e}$ by $\bar{e}_2$, the proof remains the same. Increasing a binding capital requirement increases long asset investment and decreases late period consumption.

\[ A4.2 \text{ Proof of Proposition 5} \]

This proof is to show that the expected utility of typical consumer decreases. As shown in Lemma 5, for any $\bar{e} > e^*_0$, increasing capital requirement reduces late period consumption $\frac{\partial c^*(\bar{e})}{\partial \bar{e}} < 0$. There are two possibilities for early period consumption: $d^*(\bar{e})$ decreases or $d^*(\bar{e})$ increases. If $\frac{\partial d^*(\bar{e})}{\partial \bar{e}} < 0$, then expected utility of consumer decreases for sure.

Consider the situation when $\frac{\partial d^*(\bar{e})}{\partial \bar{e}} > 0$. Suppose that for $\bar{e} > e^*_0$, $d^*(\bar{e}) > d^*_0$. Short asset investment needs to increase as well since $y^*(\bar{e}) = \frac{1}{2}(\lambda_H + \lambda_L)d^*(\bar{e})$. Define the increase in short asset investment as

$$\epsilon = y^*(\bar{e}) - y^*_0$$

The increase long asset investment is $z^*(\bar{e}) - z^*_0 = \delta - \epsilon$, since $\bar{e} - e^*_0 = \delta$.

The increase in early period consumption is $d^*(\bar{e}) - d^*_0 = \frac{\epsilon}{\frac{1}{2}(\lambda_H + \lambda_L)}$. Using the expected payoffs of safe banks under $\bar{e}$ and $e^*_0$

\[
(1 - \frac{1}{2}(\lambda_H + \lambda_L))c^*(\bar{e}) = F(z^*(\bar{e})) - K(\bar{e}) \tag{A4.2.1}
\]
\[
(1 - \frac{1}{2}(\lambda_H + \lambda_L))c^*_0 = F(z^*_0) - K(e^*_0) \tag{A4.2.2}
\]

Take difference of (A4.2.1) and (A4.2.2), the change in later period consumption is

\[
(1 - \frac{1}{2}(\lambda_H + \lambda_L))(c^*(\bar{e}) - c^*_0) = F(z^*(\bar{e})) - F(z^*_0) + K(e^*_0) - K(\bar{e})
\]
\[
(1 - \frac{1}{2}(\lambda_H + \lambda_L))(c^*(\bar{e}) - c^*_0) < F'(z^*_0)(\delta - \epsilon) + K'(e^*_0)(-\delta) = -F'(e^*_0)\epsilon \tag{A4.2.3}
\]

since $F''(z) < 0$, $K''(e) > 0$, and $F'(z^*_0) = K'(e^*_0)$.

The change in expected utility of consumer is

$$\Delta E[U] = \frac{1}{2}(\lambda_H + \lambda_L)[u(d^*(\bar{e})) - u(d^*_0)] + (1 - \frac{1}{2}(\lambda_H + \lambda_L))[u(c^*(\bar{e})) - u(c^*_0)]$$

where the first part is the change in expected utility from a higher $d^*(\bar{e})$ and the second part is the changed in expected utility from a lower $c^*(\bar{e})$.

As $u'(d^*_0) > u'(d^*(\bar{e}))$ and $u'(c^*_0) < u'(c^*(\bar{e}))$, using (A4.2.3) the change in expected utility is

$$\Delta E[U] < \frac{1}{2}(\lambda_H + \lambda_L)u'(d^*_0)\frac{\epsilon}{\frac{1}{2}(\lambda_H + \lambda_L)} + (1 - \frac{1}{2}(\lambda_H + \lambda_L))u'(c^*_0)\frac{-F'(e^*_0)\epsilon}{1 - \frac{1}{2}(\lambda_H + \lambda_L)}$$

$$\Delta E[U] < u'(d^*_0)\epsilon + u'(c^*_0)(-F'(e^*_0)\epsilon) = 0$$

The last step uses the optimality condition for consumption across periods when there is no capital
requirement. Therefore expected utility of consumer must decrease.

A4.3 Proof of Proposition 6

This is to prove that under financial opacity $\theta^*(\tilde{e})$ decreases as $\tilde{e}$ increases. This part shows that the contract savings are not sufficient to offset the decrease in payoff of risky banks. Suppose $\tilde{e}_2 > \tilde{e}_1$ and the fraction of risky banks is unchanged at $\theta^*(\tilde{e}_1)$. A higher capital requirement leads to an increase of $(\tilde{e}_2 - \tilde{e}_1)\theta^*(\tilde{e}_1)$ units of risky assets on sale in the B state. This increased asset sales will increase the return of liquidity of safe banks, which can then offer a more attractive contract than the situation in which the increased asset sales do not exist. Therefore it is sufficient to focus on the case when $\theta^*(\tilde{e}_1)$ is close to 0, i.e. not much increase in the asset sales by the risky banks and the reduction in safe bank’s contract is at the maximum.

Denote the increase in capital as

$$\delta = \tilde{e}_2 - \tilde{e}_1$$

Case (i): suppose safe banks invest all of the increased capital in the short asset. For convenience, the superscript $S$ of safe banks is omitted. The new level of short asset is $y_2 = y_1 + \delta$. The change in $d$ is

$$\Delta d = d_2 - d_1 = \frac{y_2 - y_1}{2 \lambda_H + 2 \lambda_L} = \frac{\delta}{2 \lambda_H + 2 \lambda_L}$$

The new consumption for late consumers $c_2$ satisfies

$$E[V_{ij}(C(\tilde{e}_2), \tilde{e}_2, A(\tilde{e}_2))] = F(1 + \tilde{e}_2 - y_2) - (1 - \frac{1}{2}(\lambda_H + \lambda_L))c_2 = K(\tilde{e}_2)$$

The change in the amount of resources paid to late consumers is

$$(1 - \frac{1}{2}(\lambda_H + \lambda_L))(c_2 - c_1) = K(\tilde{e}_1) - K(\tilde{e}_2) + F(1 + \tilde{e}_2 - y_2) - F(1 + \tilde{e}_1 - y_1) = K(\tilde{e}_1) - K(\tilde{e}_2)$$

Since by assumption $y_2 = y_1 + \delta$, therefore $1 + \tilde{e}_2 - y_2 = 1 + \tilde{e}_1 - y_1$.

The change in total cost of contract in terms of date 2 good in the G state is

$$\Delta = (\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L)(d_2 - d_1) + (1 - \frac{1}{2}(\lambda_H + \lambda_L))(c_2 - c_1) = \delta + K(\tilde{e}_1) - K(\tilde{e}_2)$$

Substitute into the change of risky bank payoff $\Delta E[V_{ij}^{R}]$. Also use $q_G = 1$ and multiply the contract savings by $\pi$

$$\Delta E[V_{ij}^{R}] = \pi[R_G \delta + (\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L)(d_2 - d_1) + (1 - \frac{1}{2}(\lambda_H + \lambda_L))(c_2 - c_1)] + K(\tilde{e}_1) - K(\tilde{e}_2)$$

$$= \pi[R_G \delta - (\delta + K(\tilde{e}_1) - K(\tilde{e}_2))] + K(\tilde{e}_1) - K(\tilde{e}_2)$$

$$= \delta (R_G - 1) + \delta(1 - \pi) + (1 - \pi)(K(\tilde{e}_1) - K(\tilde{e}_2))$$

$$< \delta (R_G - 1) + \delta(1 - \pi) - (1 - \pi)K'(\tilde{e}_1)\delta \quad \text{as} \quad K''(e) > 0$$

$$< \delta (R_G - 1) + \delta(1 - \pi)(1 - K'(\tilde{e}_1)) < 0$$

since $\pi R_G < \pi R_G + (1 - \pi)R_B < 1$ and $K'(\tilde{e}_1) > 1$. So if safe banks invest all increased resources
\( \delta \) in the short asset, the contract savings are not sufficient to make risky banks break-even.

Start from case (i) and use it as a benchmark. Suppose safe banks decrease short asset investment by \( \epsilon \). The new decision of short asset is \( \hat{y}_2 = y_2 - \epsilon \). As \( \hat{y}_2 < y_2 \), \( \hat{z}_2 = z_1 + \epsilon \) and \( F' (\hat{z}_2) < F' (z_1) \). The change in the total resources paid to early consumers is

\[
(\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L)(\hat{d}_2 - d_1) = \hat{y}_2 - y_1 = y_2 - \epsilon - y_1 = \delta - \epsilon
\]

The consumption for late consumers \( \hat{c}_2 \) is given by

\[
E[V_{ij} (\hat{C}(\hat{e}_2), \hat{e}_2, (\hat{y}_2, 0, \hat{z}_2))] = F(1 + \hat{e}_2 - \hat{y}_2) - (1 - \frac{1}{2}(\lambda_H + \lambda_L))\hat{c}_2 = K(\hat{e}_2)
\]

The change in resources paid to late consumers is

\[
(1 - \frac{1}{2}(\lambda_H + \lambda_L))(\hat{c}_2 - c_1) = K(\hat{e}_1) - K(\hat{e}_2) + F(1 + \hat{e}_2 - y_2 + \epsilon) - F(1 + \hat{e}_1 - y_1)
\]

Substitute into the change of risky bank’s payoff \( \Delta E[V_{ij}^R] \) and use \( q_G = 1 \)

\[
\Delta E[V_{ij}^R] = \pi[R_G \delta + (\frac{1}{2} \lambda_H + \frac{1}{2} \lambda_L)(\hat{d}_2 - d_1) + (1 - \frac{1}{2}(\lambda_H + \lambda_L))(\hat{c}_2 - c_1)] + K(\hat{e}_1) - K(\hat{e}_2)
\]

\[
= \pi[R_G \delta - (\delta - \epsilon + K(\hat{e}_1) - K(\hat{e}_2)) + F(1 + \hat{e}_2 - y_2 + \epsilon) - F(1 + \hat{e}_1 - y_1)] + K(\hat{e}_1) - K(\hat{e}_2)
\]

\[
= \pi R_G \delta - \pi(\delta - \epsilon - \pi(K(\hat{e}_1) - K(\hat{e}_2))) - \pi(F(1 + \hat{e}_2 - y_2 + \epsilon) - F(1 + \hat{e}_1 - y_1))
\]

\[
+ K(\hat{e}_1) - K(\hat{e}_2)
\]

\[
= \delta(\pi R_G - 1) + \epsilon(1 - F'(1 + \hat{e}_2 - y_2 + \epsilon)) + \delta(1 - \pi)(1 - K'(\hat{e}_1)) < 0
\]

As \( F''(z) < 0 \), \( F(1 + \hat{e}_2 - y_2 + \epsilon) - F(1 + \hat{e}_1 - y_1) < \epsilon F'(1 + \hat{e}_2 - y_2 + \epsilon) \), therefore the change in payoff

\[
\Delta E[V_{ij}^R] < \delta(\pi R_G - 1) + \epsilon(1 - F'(1 + \hat{e}_2 - y_2 + \epsilon)) + \delta(1 - \pi)(1 - K'(\hat{e}_1)) < 0
\]

since \( \pi R_G < \pi R_G + (1 - \pi)R_B < 1 \), \( K'(\hat{e}_1) > 1 \) and \( F'(1 + \hat{e}_2 - y_2 + \epsilon) > 1 \). So for any combination of short asset and long asset chosen by safe banks, the contract savings are not sufficient to make risky banks break-even. In the new banking equilibrium, the fraction of risky banks must decrease \( \theta^*(\hat{e}_2) < \theta^*(\hat{e}_1) \).
Appendix B

B1 Corporate Credit Default Swap Data of U.S. Banks 2005-2009

The graph plots levels of 5-year corporate credit default swap in basis points of major U.S. banks. Although banks had considerably different subprime exposures from 2005 to 2007, the perceived level of risk was similar for all banks.

Figure 3: 5-Year CDS Data of U.S. Banks 2005-2009

B2 Numerical Simulations

Utility function is \( u(c) = \log(c) \). The long asset in Example 4A is represented by \( F(z) = 1.2z - \frac{z^2}{12} \), for \( z \in [0, 1.2] \). Therefore \( F'(z) = 1.2 - \frac{z}{6} \) and \( F''(z) = -\frac{1}{6} < 0 \).

The investment opportunity outside of the banking sector is represented by \( F_O(\hat{w}) = 1.2\hat{w} - \frac{\hat{w}^2}{12} \), for \( \hat{w} \in [0, w] \). Therefore the cost of capital is

\[
K(e) = (1.2w - \frac{w^2}{12}) - (1.2(w - e) - \frac{(w-e)^2}{12})
\]

Use \( w = 0.6 \), the cost of capital is

\[
K(e) = 0.69 - 1.2(0.6 - e) + \frac{(0.6-e)^2}{12} = -0.03 + 1.2e + \frac{(0.6-e)^2}{12}
\]

Therefore \( K'(e) = 1.2 - \frac{0.6-e}{6} = 1.1 + \frac{e}{6} \) and \( K''(e) = \frac{1}{6} > 0 \).